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FROM NASH EQUILIBRIUM TO DARWINIAN EQUILIBRIUM IN THE EVOLUTIONARY GAME THEORY

Abstract

The main purpose of this paper is to delineate the contemporary development of Game Theory as regards to the evolutionary approach, whose scientific value is due to its interpretative capability of real phenomena both in biologic-genetic and in economic field. In the paper it is shown that these studies, which are based on the idea of "evolutionar(il)y stable strategy" and on "replicator" dynamics, implied the overcoming of the deadlock situation in which Game Theory had come because of the multiplicity of Nash equilibria, in spite of the numerous attempts to analyze a selection process of this type of equilib-rium by studying its refinements. In the paper there are also some references to the main mathematical aspects of evolutionary games and to interesting applications of this approach to economic contexts.

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1. Introduction

From a non superficial analysis of the games that in literature are qualified as "dynamic", it emerges that their dynamic connotation resides only in the variability of the players actions in the different subsequent states. Such actions nevertheless are a consequence of combined strategies pre-existing the same dynamics. In order to understand such observation it is enough to think about a game represented in the exten-

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sive form, that can be seen as a dynamic game if time is considered in order to identify the passages through the various "knots" of the "tree" representation, according to a sequence deriving by a "first move" and also bound to a pre-defined scheme of possible consequent actions even in a multi-alternative way. We could also say that in the dynamic games the elements in comparison, that is to say, in conflict, are perfectly rational individuals, while in the evolutionary games the conflict takes place among strategies, that acquire the role of "players."

The complexity and the elevated variability of the economic contexts to which often and in more and more intense way the Theory of Games is applied, have induced the researchers of "intelligent"[†] choice methods, to widen the elementary dynamic structure of the aforesaid games.

Before the interesting developments of the evolutionistic approach, the literature on the Theory of Games had reached a deadlock situation, due to the problems of analysis correlated to the multiplicity of Nash equilibria and to the numerousness of its refinements finalized to the individualization of a method for "choosing" an equilibrium in the inside of that multiplicity. The problem of the choice of Nash equilibria has only apparently been resolved by the research of its refinements, in fact, once seen the elevated number of the latter, every Nash equilibrium in a context with multiple equilibria ended up finding a reason to exist.

The individualization of the equilibrium toward which the system places itself dynamically and spontaneously is the result of an analysis that keeps in mind how the gen-erations of rational and intelligent subjects modify their own beliefs and rules: the equilibrium to which we come to, allows to observe (as to say, ex post) the validity or the non-validity, as well as the consistence, of the strategies that the subjects themselves have formulated according to their beliefs as well as their objectives. All this can be studied in a rigorous way if we use a model based on an evolutionistic approach, in which the "evolution" takes the

[†] Keep in mind this interesting definition: the Theory of Games is an area of Mathematics that studies models of con-flict and cooperation among rational and intelligent agents. With the term "rational" it is intended to characterize every agent that maximizes his objective function (expected utility, profit). With the term "intelligent" it is intended to attribute to the agent the ability to become aware to interact with other subjects that are rational and intelligent and, therefore, it is intended to give him the propensity to take decisions according to the suggestions deriving not only by a pure individualism, but also from the interaction with the others ("strategic interaction").

form of "cultural evolution."

In Game Theory, by the adjective "evolutionary" it is intended to characterize the approach that has had origin from a stream of research in biological field, primarily fed by geneticists, whose studies were finalized to the justification of some specific equilibria within biological and natural phenomena. Also in regarding to the classical Game Theory it can be said that it has broadly been applied to biology, even though it had been conceived by Von Neumann and Morgenstern (Von Neumann, Morgenstern, 1944) for the study about the economic behaviours.

One of the fathers of the Evolutionary Game Theory is John Maynard-Smith, who, in 1972, published an article entitled "Game Theory and the evolution of fighting" (Maynard-Smith, 1972), which it is deemed to contain the roots of the modern approach of the theory of games. His contribution was subsequently improved in cooperation with George Price (1973). To say the truth it must be said that these articles had been preceded by the work of the geneticist Fisher (1930) who had already been dealing with the theme of the natural selection, with characteristics tightly similar to the one treated by Maynard-Smith.

Another meaningful contribution, previous to the development of the evolutionistic literature of games, has been furnished in 1969 by the researcher David Lewis, who correlated the classical Theory of Games to implicitly cooperative behaviours of people speaking the same language, creating more than one interesting and fascinating hint for the analysis of linguistic processes to be considered as evolutionary games.

In this synthesis of the scientific background that precedes the contemporary developments of the Theory of Games, it is not possible not to make reference to the thought of Charles Darwin (1859). In fact it is sure that the Darwin's ideas about the natural selection and on the biological evolution have influenced the economic theory, and it cannot be said if it has been done in a direct or indirect way, in at least two great fields of study: the neoclassic theory and the evolutionary game theory. Within the neoclassic theory the processes of natural selection are those that allow the optimizing agents, being perfectly rational, to survive to the risks of exclusion from the market, whose vic-tims are the least rational subjects or those who are not able to operate efficient choices both in a technical sense and in an economic sense. In this kind of studies, evolutionistic type reasoning have been, therefore, used for defending the hypothesis, rather unrealistic, of the existence of perfectly rational subjects that would apply the principle of individual rationality (cost-benefit analysis) with instrumental mathematical ability. There is to say that the evolutionary approach has also been used for explaining, contrarily, irrational behaviours like those that produce the speculative bubbles in the financial markets, on the base of analysis of the "flock" behaviour, that is, of the behaviour produced by the adjustment to the choices of the majority. The influence of Darwinian ideas on the Evolutionary Game Theory is more evident and profitable, and it can be interpreted as the contribution that Darwin would have had "to pay" to the economic science in order to cure one debt of his, considering that he himself affirms to have thought for the first time about the natural selection following the reflections induced by the reading of the *Essay on the Principle of Population* published by Malthus in 1798.

2. The Theory of Evolutionary Games

The Theory of Evolutionary Games has developed beginning from the Eighties after the publication, in 1982, of the book *Evolution and the Theory of Games* by J. Maynard-Smith and it has the great merit to have given value and significance to the concept of Nash equilibrium, especially in contexts in which a multiplicity of such equilibria are present.

In fact Nash himself has observed how the notion of equilibrium defined by him becomes more reasonable if it is seen as the result of a dynamic process based on the learning deriving from a succession of phases not perfectly rational: a Darwinian dy-namics of a cultural type could be a model of such process. It can be said more precisely that, according to the cooperative or non-cooperative results of the interaction of rational subjects, a cultural type selection process is determined owing to the fact that, in each following stadium, the behaviours or strategies that have produced more advantageous results are adopted by a greater number of subjects creating, in this way, a modified cultural configuration in comparison to the initial one. When there are more possi-ble equilibria, the one characterized by the so-called "evolutionary stability", will be able to affirm itself, revealing to be resistant to following mutations.

In other words, among the strategies of a population of players, some will be des-tined to disappear, while others will spread, and this will happen according to their "reproductive success", to their *payoff*. The strategies therefore undergo an evolutionary process that conducts, through the "winning" strategies, to the selection of an equilibrium.

The Theory of Evolutionary Games studies this type of selection hypothesizing a context in which limitedly rational players are present and they are casually extracted from a wide population. The hypothesis of limited rationality is to be intended as hypothesis of "adaptive behaviour", which is not based on rigorous reasoning, but on the learning deriving from the repetition of the game and on the consequent adaptation to the strategies that have implicated the greatest relative success.

The strategies adopted by the players are not conceived as the result of the rigorous application of the principle of individual rationality, but as schemes of behaviour acquired through the genetic and cultural transmission. In other words the concept of hu-man rationality is replaced by the evolutionary stability. The advantage inherent in the evolutionistic approach to the Theory of Games, which can guarantee a sure success, is in the fact, as John Maynard-Smith says, that in the reality, there are many reasons to expect that the population of the players evolves toward stable states and there are, instead, many doubts about the rationality of human behaviour.

In the models finalized to the analysis of evolutionary contexts the dynamic connotation, predominantly not linear, becomes richer compared to the classical formulation within which, even if it could be referred to dynamics, anyway this was based on predetermined schemes. In the Theory of Evolutionary Games the change becomes protagonist and it involves the number of the individuals, the individual and group strategies, the number of the competitions that must be faced, always producing new configurations, from which taking, again, the moves for following games.

Exactly for all these aspects it is affirmed that paradoxically the Theory of the Evolutionary Games could better apply to the economic behaviour than to the animal behaviour to which had initially been designated, as paradoxically, according to Maynard-Smith, the classical theory of games is better applied to biology and to the animal behaviour rather than to the economy for which it had been thought by von Neumann and Morgenstern.

3. A fundamental element of the Theory of Evolutionary Games: the concept of evolutionarily stable strategy

In the development of Evolutionary Game Theory a fundamental stage is marked by the rigorous definition of a new type of strategy, characteristic of competitive contexts in evolution: the evolutionary stable strategy or the evolutionarily stable strategy (ESS). These strategies are such that their results cannot be improved by the realization of any other alternative strategy. The definition of ESS implicitly contains the punitive component of the strategy itself, implicating the loss of part of the *payoff* for every possible deviation. Insofar, if the population of the players chooses an ESS, the latter is destined to be the winning strategy, firmly dominant. The idea of the ESS was originally systematized by Maynard-Smith and Price in their essay (1973), within the presentation of a famous evolutionary game, the one undertaken between hawks and doves for the appropriation of an unique resource.

From the model of the aforesaid game it is possible to enucleate a definition of ESS and a theorem that expresses the sufficient conditions for such type of strategy, obviously suitable to the context of the game of conflict between hawks (aggressive) and doves (cooperative and not aggressive). To understand the content of this definition and of this proposition, both fundamental for the analysis of the evolutionary games, it is opportune to identify the players that operate according to a strategy with the strategy itself.

Definition of ESS. A strategy "*a*" is defined evolutionarily stable if, given a context in which almost the totality of players is composed by individuals "*a*", no mutant strategy "*b*" has the possibility to invade the population of players, by increasing the number the "*b*" players and by reducing the number of the "*a*" players.

Theorem (sufficient conditions for an ESS). A strategy "*a*" is ESS if one of the following conditions is verified:

- a player "*a*" obtains, meeting with another "*a*" a greater *payoff* in comparison to the one obtained by the meeting of a "*b*" player with an "*a*" player;
- an "*a*" player and a "*b*" player get equal *payoffs* from a meeting with an "*a*" player, while, meeting with a "*b*" player, an "*a*" player gets a

greater payoff than a "*b*" player.

The notion of equilibrium that is correlated to the determination of an ESS is an extension of the original concept of Nash equilibrium to evolutionary contexts. Particularly, referring to a game with two players belonging to the same population, the following proposition can be proved.

Proposition. An ESS is an action a^* belonging to the whole strategies of the two players, such that

- (a^*, a^*) is a Nash equilibrium;
- for every mutant strategy $a \neq a^*$, that is a best replay to a^*, the deriving *payoff* from the couple of strategies (a , a) is inferior to that deriving from the couple of strategies (a^*, a).

The criterion of evolutionary stability, that originates from the definition of ESS, with the purpose to select the "best" equilibrium is surely more restrictive and, therefore, it is more effective than the various *refinements* of Nash equilibrium. It introduces, however, some application limits and some lacks: the first ones can be found in the case of non-symmetrical games in which the players belong to two different populations (the preceding definition supposes that the two players belong to the same population); the lacks refer to the fact that the criterion is not inferred from an analytical model that formalizes the process through which we come to the stable equilibrium and therefore it doesn't furnish a method of investigation for the study of the evolutionary process that characterize the dynamics of the game.

The dynamic analysis based on the concept of "replicators", formulated in mathematical terms by Taylor and Jonkers (1978), as well as by Zeeman (1980), tries to give answer to this type of demand which is not satisfied by the concept of ESS alone, without being in antithesis with it, but including it as particular case: in fact the stable steady states of the dynamic models with "replicators" (*replicator dynamics*) coincide with the equilibria induced by the choice of ESS in the case in which at most two pure[*] strategies exist, while they differ whenever pure strategies are more than two.

[*] It is said that a player adopts a pure strategy when he directly chooses one of the moves available, it is said, instead, that he adopts a mixed strategy when his choice concerns a combination of the available strategies after having attrib-uted to each one of them the probability of it to be carried out.

The dynamic analysis of the "replicators" appears to be very interesting, being, in the actual state of the mathematical literature on the evolutionary games, the most complete method to investigate, in a rigorous way, the evolutionary processes that lead to stable equilibria in situations of strategic interaction among populations of "intelligent" sub-jects. Such mathematical approach identifies the concept of "replicators" with that of *pure strategy*, whose diffusion within the population depends on the "reproductive advantage" (Darwinian fitness) gotten from it. The latter is positively correlated to the dif-ference between the current *payoff* of the pure strategy and the average *payoff* of the population. The higher becomes the advantage of the considered pure strategy, the more numerous copies of the same will be created.

4. The mathematics of the evolutionary games

The theoretical elements delineated in the preceding section underline that, in the actual state of the literature, the attempts to mathematically formalize the evolutionary games are essentially two: the one based on the concept of "evolutionarily stable equilibrium" and the one that analyzes the dynamics of the "replicators." In this section the essential lines of both types of mathematical approaches are traced and commented.

The analyses that stimulate the concept of ESS often have descriptive characteristics, even if sustained by a strong logical-deductive rigor and by hypotheses and definitions formulated coherently to the mathematics of the traditional Theory of Games. In other words, the concept of "evolutionarily stable equilibrium", once defined, is used in the same way of Nash's concept of equilibrium, in comparison to which it is more restrictive, seen the addition of a further condition.

The definition of evolutionary game in strategic form, for instance, is not very far from that of a non-repeated or repeated classical game type. In fact, to define an evolutionary game it is enough to furnish the list, with the relative definitions, of the number of populations of players involved; the list of the strategies of the players (belonging to the same population or different populations); the list of the *payoff* functions of each player, depending by its own strategy and by the rivals' strategies in presence of a note distribution of probability of these latter.

The differences from the traditional theory are due to the interpretation of the role of these constitutive elements of the evolutionary game: in fact the concept of "player" of the classical theory of the games identifies itself with that of "pure strategy" that is a characteristic of the population which the player himself belongs to, as it happens in the game between hawks and doves, where the hawk player, belonging to the population of the hawks, identifies itself with the "hawk" strategy and it is the same also for the dove player: the conflict is seen as the conflict among strategies. Likewise the *payoff* functions, expressing for each strategy (player) the relation between the combinations of it-self with others and the consequent advantage, acquire an evolutionary meaning, in the sense that they will have to give measure, even indirectly, of the probability with which the strategy (player) will result "winning": it will survive the conflict, subsequently re-plying itself. For their nature such functions are included in the definition of ESS, as it can be inferred from what is exposed in a simple way in the preceding section, where the conditions to be verified so that a strategy is evolutionarily stable, are expressed as conditions on the *payoffs*.

Also the formal analysis of these payoff functions is not far from the traditional one and often they are objective functions in problems of constrained optimization.

In literature different criteria of evolutionary stability are present but the one based on the concept of ESS, originally formulated by Maynard-Smith and Price (1973), is the most effective one, even if more restrictive (Dufwenberg, 1997).

As in regards to the replicator dynamics, already mentioned in the two preceding sections, the mathematical analysis is based on the resolution of differential equations (or finite difference equations) whose number is equal to the number of the replicators (pure strategies) and whose unknown variables are the "reproduction differentials", that is the differentials, calculated in dependence on time, of the ratio of the number of players of a strategy and the number of players of other strategies. This ratio depends, in every instant, on the value of the ratio itself in the preceding instant and on the relative variation of the *payoff* level. The variations of the ratio are positively correlated to the relative variation that measures the "reproductive advantage", since the number of players that adopt the strategy in examination grows, in comparison to the number of those who don't adopt it, when the "reproductive advantage" grows.

The mathematical analysis of such differential equations goes toward the determination of stable steady states, that, as it has been verified, not always coincide with the equilibria induced by ESS. The replicator dynamics, in fact, allows to individualize evolutions of other nature, like those that can lead, for instance, to stable cycles and not necessarily to univocally determined stable equilibria.

Even if this approach based on the differential equation analysis includes a greater number of study cases, it is evident that none of the two mathematical methods described in this section has an aetiological value, since, it is not able to explain the causes of the evolutionary phenomena that lead to stable equilibria.

This consideration takes us to the conclusion that the Evolutionary Game Theory needs a reformulation that systematizes in a rigorous way all the elements, including in itself the aetiology of the events that it describes.

5. Applications of the Evolutionary Game Theory to Economics

The effectiveness of the evolutionistic approach to the analysis of complex and global contexts as those that are object of study of the contemporary Economics, has implicated a great success of the Theory of Games among the economists, superior to that, not universally shared, that the same Theory, in its traditional version, had already had in the Eighties, when many economists granted to it the role of instrument of a great utility for Economic Sciences (Mattoscio, Pagliari 1990, pp. 10-12). This success sustains the forecast of many researchers, among which Gintis (2000), who deem that the Theory of Games could assume the role of universal language for the Behavioural Sciences. In this role the classical approach could be considered a normative version of the theory, pointing out the choices that rational people should make, while the evolutionistic approach could be the descriptive version, for studying the decisional processes that individuals really undertake.

The interest aroused by the evolutionistic theory of games has stimulated many applications in different disciplines. Here after it is intended to mention the applications in the economic field and in analogous fields.

In Knowledge Economics the *learning evolving* is studied on the base of evolutionary processes in contexts in which the learning is analyzed through genetic algorithms.

The Moral Philosophy and the Political and Social Sciences use evolutionistic models for the study of such important themes as the distributive equity, the social justice, the altruistic behaviour in the human kind, the necessity of private ownership, the international politics. Equally interesting are the applications that study the strategic behaviour of the tax-payers, finalized to analyze how the social disapproval towards the tax-evader might implement a correct behaviour in relation to the fiscal duties on behalf of the collectivity.

References

Darwin C. (1859), *On the Origin of Species by Means of Natural Selection, or the Pres-ervation of Favoured Races in the Struggle for Life*, John Murray, Londo.; Reprinted in Mayr E. (1964), ed., *On the Origin Of Species: A Facsimile of the First Edition*, Massa-chusetts, Harvard University Press, Cambridge.

Darwin C. (1873), *The Descent of Man*, D. Appleton and Company, New York.

Dufwenberg M. (1997), *Some relationships between evolutionary stability criteria in games*, Economic Letters, 57, 45-50.

Fisher R. (1930), *The Genetic Theory of Natural Selection*, Clarendon Press, Oxford.

Gintis H. (2000), *Classic versus evolutionary game theory*, Journal of Consciousness Studies, 7(1-2), 300-304.

Lewis D. (1969), *Convention*, Harvard University Press, Cambridge.

Mattoscio N., Pagliari C. (1990), *Equilibri economici non cooperativi e teoria dei giochi*, Cedam, Padova.

Maynard-Smith J. (1972), *Game theory and the evolution of fighting, in On Evolution* (J. Maynard-Smith , ed.), Edimburgh University Press, Edimburgh.

Maynard-Smith J. (1982), *Evolution and the Theory of Games*, Cambridge Univ. Press, Cambridge.

Maynard-Smith J., Price G. (1973), *The logic of animal conflict*, Nature, 146, 15-18.

Taylor P.D., Yonker L.B. (1991), *Evolutionary stable strategy and game dynamics*, Neurobiology, 88, 4433-4437.

von Neumann J., Morgenstern O. (1944), *Theory of Games and Economic Behaviour*, Princeton University Press, Princeton N.J..

Zeeman E. C. (1980), *Population dynamics from game Theory*, Global theory of dynamical systems, 471-497, Springer-Verlag, London