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INTRODUCTION

to

THE SPECIAL ISSUE OF GLOBAL & LOCAL ECONOMIC REVIEW

Surrogates, Humbugs and other Aggregate Parables.
Half-a-Century of the 'Surrogate Production Function'

In honour of
Anwar Shaikh

That forty years have passed since a young Anwar Shaikh firsts shook the complacency of the neoclassical establishment is a simple fact, yet like all such simplicities of academic life, hard to fathom. Shaikh's bolt was not from the proverbial 'blue'; those of us who had the privilege to study under the influence of the Cambridge *maestros*, Joan Robinson and Richard Goodwin, Nicky Kaldor and Luigi Pasinetti, Richard Kahn and Geoff Harcourt[1], were almost lulled into a different kind of complacency – that which was associated with an acknowledged intellectual victory.

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[1] No one in my generation, to the best of my knowledge, was directly supervised by the great Piero Sraffa, but everyone knew him, talked to him, and his general intellectual spirit permeated the life of the doctoral students and his works were the most discussed topics in the room which was allocated to us, in the Sidgewick site.

Sraffa's *magnum opus*, *Production of Commodities by Means of Commodities* (PCMC), was on all our desks, in our bags, on our shelves – and so was Harcourt's wonderful 'blow-by-blow' account of the *Cambridge Controversies in the Theory of Capital*. Mario Nuti was giving his brilliant lectures on *Capital and Time*, outlining his fundamental results on what eventually came to be called the *Truncation* or *Neo-Austrian* Theorem by John Hicks. The great architect of the neo-Walrasian resurgence was himself reminding us that there was once a J.R. Hicks, who later wrote as Sir John Hicks but, as a result of the Sraffian impetus had begun to refer to himself as plain and simple 'John Hicks'[2].

We felt that it was only a question of time before Paul Samuelson's *Economics* would be replaced by *Introduction to Modern Economics* by Joan Robinson and John Eatwell. We were already mesmerised by Richard Goodwin's remarkable 'principles' lectures in book form, *Elementary Economics from the Higher Standpoint*.

Amartya Sen came to give a Faculty Seminar, introduced by Frank Hahn, to a packed room of doctoral students, a majority of whom were young Italians, but both John Eatwell and Piero Garegnani were also present. Hahn remarked, in passing, that 'bees had been attracted to honey', when he saw the packed room, referring to the title of Sen's talk: *On the Monotonicity of the Value of Capital*[3].

It was to this milieu that the youthful – we did not know that it was a young 27/28 year-old who had written it[4] - Shaikh's HUMBUG arrived, first passed on to one of us, the doctoral students, by Geoff Harcourt.

[2] In my own contribution to this Special Issue of G&L ER I take the liberty of quoting a significant passage from the letter John Hicks wrote, on 3 September, 1960, Piero Sraffa, after his initial reading of PCMC.

[3] Later published in the Journal of Economic Theory, 1975, as *Minimal Conditions for the Monotonicity of Capital Value*. We were, of course, also well-schooled in Sen's contribution in the A.K. Das Gupta Festschrift, where the hapless Subhuti was trying to reach a kind of nirvana, by conversing on capital theory, on the strength of his knowledge of the subject, having read Harcourt's *JEL Survey*, which became, eventually the famous book.

[4] We had, of course, already struggled with the 1967 QJE paper by Joan Robinson and K.A, Naquvi and were, in a sense 'prepared' for HUMBUGS!

Ours was a shadowy existence, like figures in a *Nō* play, but the manuscript by Shaikh, not yet published, spread like the proverbial wildfire and lit our shadows with incandescent light, for a while.

We[5] felt, then, that it was the icing on the cake of Cambridge (UK) dominance and the sure decline and demise of the neoclassical hegemony. I myself recall a conversation with Geoff Harcourt, in around May, 1973, who reported that Joan Robinson had told him that the 'electrifying intellectual atmosphere', in the Faculty, at that time, reminded her of the halcyon days of the 'Keynesian revolution' of the 1930s.

How wrong we were!

Now forty years later we are surrounded by colleagues who teach macroeconomics from textbooks – for example Romer's Advanced Macroeconomics[6] - routinely formalizing an aggregate production function in its discredited Cobb-Douglas form, as if the HUMBUG was just a great deal of water poured over a duck's back.

Anwar Shaikh has, of course, moved on – yet the originality displayed in the path-breaking RESt paper of 1974, informs his fundamental philosophy and methodology of economics: the search for foundational rigour, tempered by empirical sense.

I am sure I express the true and sincere sentiments and beliefs of all the contributors to this Special Issue that a new generation of graduate students, all over the academic world, will be inspiredx by Shaikh's recently completed *magnum opus*. The academic community in economics is, surely, less open now – despite the demoralizing economic situation all around us. We can only hope that these events will loosen the iron hold of a variety of orthodoxies in economic

[5] Guglielmo Chiodi, a distinguished contributor to this *Special Issue*, is my witness to this record and impressions; he and I were exact contemporaries at Cambridge and, indeed, we both had the pleasure and privilege of being supervised by Richard Goodwin. Chiodi was at Peterhouse and I at King's.

[6] Both in Cambridge and the New School for Social Research – and, of course, everywhere else where newclassical dominance is supreme and complete.

theory – particularly various forms of New Keynesian economics and the virulent version of so-called newclassical economics – so that fresh, youthful, minds will be allowed to roam freely and explore adventurously, to discover the many gems in Shaikh's mature reflections on economics from a radical Political Economy perspective.

It is, for me at least, as if I was privileged to re-read Maurice Dobb or Shigeto Tsuru, all over again, in the equally enlightened company of Anwar Shaikh.

It is not for me to summarise the original contributions by the set of distinguished 'Shaikh admirers'. However, let me make a few marginal (*pace* Sraffa – but this is, as pointed out in the Preface to PCMC, a 'spurious' margin) comments on some of the papers.

Firstly, the lead paper in the issue is, naturally, that by our common friend and teacher, Geoff Harcourt. In it Geoff outlines the harsh experience Shaikh's HUMBUG encountered, when he sought to publish it in an reputable orthodox Journal. Yet, I remain puzzled! Surely, both the Economic Journal and the Quarterly Journal of Economics[7] were still fairly relaxed and receptive to Cambridge (UK) critiques of a variety of neoclassical constructs, as well as the Oxford Economic Papers? Amit Bhaduri's Marxian Capital theoretic paper had been published in the Economic Journal in 1969 and Mario Nuti's influential contribution to capital theory in 1970.

Why Shaikh published his fundamental contribution in the RESt– or even approached it in the first place, as a first choice, if he did – remains, for me, and I am sure, many others, a mystery. Why did he not approach the EJ, QJE or the OEP, first – or did he, and was the RESt a fourth or 'lower' choice?[8]

[7] Indeed, I had myself published in the QJE in 1975!

[8] I am motivated to ask this question by the story of the 'withering rejection' Lucas claims to have received from a leading economic Journal, to which he had first submitted his classic on Expectations and the Neutrality of Money, eventually published in an early issue of JET. This story is narrated by Lucas p. 10 of the *Introduction* to his Studies in Business-Cycle Theory.

Harcourt's brilliant and sympathetic recollections remains silent on this issue, but it is a rich source of those who are interested in the vicissitudes of intellectual history, particularly when non-orthodox scholars try to report rigorous results challenging the complacency of one or another form of orthodoxy[9].

My only other comment on the contributions is almost trivial, bordering on the banal!

All of the contributions, with the exception of the brilliantly original one by Prabhat Patnaik, refer to, or start from, the Shaikh HUMBUG, either in their theoretical ruminations (as in Chiodi, Opocher & Steedman, Bucciarelli, Mattoscio & Alessi, Felipe & McCombie and Velupillai) or in their elegant empirical exercises (as in Jayati Ghosh and Godin & Kinsella) or, also in the outstanding 'hybrid' contribution by Fredholm & Zambelli. Professor Patnaik alone, in his characteristically original way, pays homage to Shaikh from a perspective which is also treasured by the *honoree*: a Marxian point of view.

This compact – yet intellectually rich – collection in honour of Anwar Shaikh is a tribute to the originality, audacity and courage shown by a scholar of impeccable intellectual integrity. Shaikh, like Myrdal, Tsuru and Dobb, who I consider his true peers, has always been swimming against the current, almost salmon-like. It so happens, he is also a keen and enthusiastic 'fisherman'; that the fate of seasonal salmons has not been his lot is a testimony to his incredible resilience and tenacity, in the face of relentless adversity.

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[9] Here, too, my own experience is a kind of testimony. The review article of the above Lucas book my former colleague Fitoussi and I wrote for the Journal of Money, Credit and Banking was, at first, blocked by the editor. Only a determined intervention by Don Patinkin saw to it that it was, eventually, published in that journal. On the 'other side', when I, in my capacity as the Book Reviews Editor of the Journal of Economic Dynamics and Control, 'commissioned' Art Devany to write a review article of Sargent's Ryde Lectures, Bounded Rationality in Macroeconomics, and he produced an excellent, but critical, essay, the Editor of the Journal was reluctant to go ahead with publication without a drastic revision. I refused to 'compromise' and even offered my resignation if it was not published 'as is', in which form it finally appeared. Editorial hegemony in fostering the dominance of an orthodoxy permeates the academic profession and the so-called objectivity of peer-reviewing is simply a travesty of the true situation – as poignantly described by Geoff Harcourt, of the fate that befell Shaikh's HUMBUG.

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Geoffrey Colin Harcourt*

THE IMPORTANCE OF HUMBUG IN THE CAMBRIDGE - CAMBRIDGE CONTROVERSIES IN CAPITAL THEORY

Abstract

The article outlines the harsh experience Anwar Shaikh's HUMBUG encountered, when he sought to publish it in an reputable orthodox Journal. This is also a source for those who are interested in the vicissitudes of intellectual history, particularly when non-orthodox scholars try to report rigorous results challenging the complacency of one or another form of orthodoxy. On this regard, the article further recalls the editorial hegemony in fostering the dominance of an orthodoxy which permeates the academic profession and the so-called objectivity of peer-reviewing as occurred to Shaikh's HUMBUG.

JEL Classification: A11, B24, B31.

Keywords: Anwar Shaikh, Robert Solow, Review of Economics and Statistics, Law of Production and Law of Algebra.

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1. Introduction

I am delighted and honoured to contribute an essay to the Special Issue of the *Global and Local Economic Review* in honour of Anwar Shaikh's contributions, and especially to honour 40 years on from the publication of his wonderful HUMBUG article, "Laws of production and laws of algebra: the Humbug production function" in the February 1974 issue of *The Review of Economics and Statistics*.

Anwar and I have been friends since the publication of my 1969 *Journal of Economic Literature* survey article, "Some Cambridge controversies in the theory of capital". Legend has it that, as a graduate student at Columbia (the 1974 article originated in his Ph.D dissertation, "theories of value and theories of distribution" (1973)), Anwar had a dog-eared copy of the survey in his back pocket as the repository for an oft read back up to his remarkable doctoral dissertation on issues in the controversies. We met when I visited the New School. Once he very kindly had me stay in his New York apartment, squeezing me in beside the Pakistani radicals seeking refuge from police persecution back home, who were his long-staying guests. As a fine Marxist scholar *and* activist, *Praxis* was Anwar's middle name.

I have since read with admiration many of his outstanding contributions in which he combines great technical skills with deep understanding of the conceptual bases and history of our subject, presenting his findings with the clarity and passion that only those who are on top of all aspects of their subject are capable of.

2. Bob Solow's ill-tempered and mistaken response to Anwar Shaikh

In this note I concentrate on his 1974 *Review of Economics and Statistics* paper, Bob Solow's ill-tempered and mistaken response to it, Solow (1974), and Anwar's development of the original ideas and his response to Solow's criticism in Humbug II, Anwar's chapter in Ed Nell's 1980 volume, *Growth, Profits and Property. Essays in the Revival of Political Economy*[1]. What optimists we all were then!

[1] As well as the Humbug papers, I especially admired his writings on Ricardo justified, the 93 percent

Let me first note the disparity in the lengths of the gestation periods between the submission and acceptance of Anwar's and Bob's papers:

June 1, 1972 – March 28, 1973 (Anwar); March 23, 1973 – March 28, 1973 (Bob).

Moreover, I understand that Solow insisted that his "comment" be published alongside Anwar's article, in itself a sensible suggestion, but that Anwar not be allowed to respond. I regard this as uncharacteristically poor treatment of a young scholar by a well-established one. Indeed, it is way off the regression line of Solow's well-known and rightly admired encouragement of young scholars.[2]

3. The 'vision' of the processes at work in capitalism

Apart from the technical elegance and ingenuity of Anwar's analysis, he was one of the first participants in the controversies to put the technical analysis within their proper conceptual setting. Both Amit Bhaduri (1969) and Anwar made explicit that a fundamental issue at stake was the 'vision' of the processes at work in capitalism, of how accumulation and profits arose and were related. Anwar implied immediately (and explicitly in Humbug II) that scarcity and choice in an exchange system transferred to the sphere of production underlie both the theory and empirics of Solow's response and the practice, then and now, of the mainstream generally. J.B. Clark's theory of distribution and Irving Fisher's consumer queen drive the action through her aim to maximise her lifetime expected utility, with all other actors in the economy being but the agents to allow her to achieve this. Whereas Amit and Anwar (and Maurice Dobb, Michal Kalecki and Joan Robinson) have the alternative vision of the classical political economists and Marx, of ruthless swash-buckling capitalists (all three sub-classes) producing and accumulating, with all the other actors dancing to their tune.[3]

labour theory of value vindicated (1998), his paper on the transformation problem (1997) and his systemic analysis of the motion of capitalism through transforming the Keynesian national accounts into their Marxian counterparts (1994).

[2] I know of this from personal experience. When I was preparing my 1969 survey article, Bob sent me copious comments on the working papers I circulated on the way to the final draft.

[3] It is a nice irony that in his ninth decade the late Paul Samuelson had come to a similar view point. In an

4. Technical structures and the processes of accumulation and distribution

That increases over time in output per person and per hour at the level of the firm, the industry and the economy are the outcome of both "more" and "better" capital per person are technical facts of life which economists of all persuasion accept. Neoclassical economists further argue that the effects of "deepening" and "bettering" are separable, at least in principle (this is, after all, the conceptual basis of Solow's 1956 and 1957 articles). In contrast, Post-Keynesians, for example, Nicky Kaldor and Joan Robinson, ultimately came to argue that they were not, that the factors associated with accumulation bringing about the rise in output per person through embodiment were indissolubly mixed, see, for example, Kaldor (1959), Joan Robinson (1971). It is how the above underlying technical structure is married to the processes of accumulation and distribution that creates the impassable cleavage between the two sides.[4]

5. The Cobb-Douglas production function as a law of algebra

Solow (1957) set out an ingenious way in which to precipitate out the deepening function from the overall relationship between output per person and capital per person which contained both it and the impact of technical progress in the neoclassical version of Harrod's natural rate of growth, Harrod (1939). Solow covered himself by writing that if it were assumed that the time series data used were viewed *as if* they had come from a production function in which, under competitive conditions, factor prices were equal to their respective marginal products and which was subject to the impact of neutral technical

Address to the Bank of Italy on October 2, 1997, in which he compared the different experiences of present-day American and European economies, he said: "I lay stress on two main factors … One. In America we now operate … the Ruthless Economy. Two. In America we now have a Cowed Labor force … two features interrelated … [yet] somewhat distinguishable." Especially is this so as Anwar shows that the neoclassical claims only go through if the pure labour theory of value with regard to values and prices goes through, see Shaikh (1974), 115.

[4] Duncan Foley and Tom Michl (1999) have provided an appealing classical model to illuminate the empirical findings on which the mainstream erect their analyses and findings.

progress which raised the whole function over time, he had devised a simple way to fit statistically a function to the points so precipitated out.[5] As we know it was a Cobb-Douglas.

Anwar's criticism was to show that the function that was fitted was an algebraic identity – a law of algebra – in which regardless of how the values of the various variables were created – what processes were responsible for them – GNI would always be identically equal to the share of wages plus the share of profits. Solow's methods and results could neither refute nor confirm that a Cobb-Douglas production function was the originator of what was observed in the data.

Anwar's procedure was to show how a time series spelling out HUMBUG gave the same result – a very good fit of a Cobb-Douglas – as did Solow's adjusted data. Solow's answer, which preceded his description of his methodology quoted in footnote 5 above, was that "Mr Shaikh's article [so much for a Ph.D from Columbia when viewed from MIT] [was] based on misconception pure and simple." (121).

6. A continued criticism

Anwar joined and was joined by economists from both camps, as it were. Franklin Fisher (1971), for example, carried out a huge simulation exercise in which he showed that if factor shares in the GNI were constant over 'time', a Cobb-Douglas function fitted well even though the conditions for aggregation from individual firms' Cobb-Douglas functions to the economy as a whole were ridiculously restrictive and demanding. The fit occurred because the shares were constant, not because a Cobb-Douglas was producing the observed statistics. Henry Phelps Brown (1957) (whom Anwar delightfully refers to as P. Brown) had already discussed the short-comings of the Cobb-Douglas associated with Paul Douglas's seminal work but his setting out of the critique was rather obscure and was neglected in the literature as a result. Herb Simon also made the same critique but

[5] "The factor-share device of my 1957 article is in no sense a *test* of aggregate production functions or marginal productivity …. It merely shows how one goes about interpreting given time series if one starts by *assuming* that they were generated from a production function and that the competitive marginal - product relations apply" Solow (1974), 121, emphasis in original.

again did not have an immediate impact, see Simon and Levy (1963). The person who has stuck most tenaciously to the task of propagating and developing Anwar's insights is John McCombie, more recently in the company of Jesus Felipe see, for example, Felipe and McCombie (2013). (Felipe has also collaborated with Fisher.)

Despite all this continued and damning criticism, the mainstream goes merrily on its way, using Cobb-Douglas or its sophisticated cousins, for example, CES, in both modern macroeconomic analysis and in endogenous growth theory, a procedure that is as intellectually dishonest as the continuing use of partial equilibrium supply and demand analysis after Sraffa (1926) (we are all guilty here).

Anwar (1980), 93, points out that Solow tries to have his cake and eat it too." Having … said that his method … [led] him to conclude that even the Humbug economy is neoclassical, [he] next asserts the very opposite … he runs a [linear] regression … on the Humbug data [that] gives a very poor fit [and] a negative coefficient for his *k*. [Anwar argues] that linearity is … a convenient assumption whose applicability must be … *justified*, not … assumed." (emphasis in original).

7. Bob's question and my answer

I spent a week at the Economics Department of Tufts University in 1975 through Tom Cooley's good graces. I gave a seminar on the capital theory controversies (it was based on a paper which was the sequel to my 1972 book; it was later published in *OEP* (1976)). Paul Samuelson and Bob Solow were in the front row.[6] I had to tone down what I had said in other places where I included some cracks about the two MIT inseparables (the cracks were reinstated in the published version, I am not completely lacking in integrity).

After I had given the paper Bob quizzed me as to why I was so agin marginal productivity approaches. He asked: suppose you were a business person and were thinking of employing an extra person?

[6] They had come to Tufts the week before, having mistaken the date the seminar would be on, appropriately, April 1.

Would you not do so if you expected the extra revenue so gained to exceed the extra wage paid? I said I supposed I would but, being rather non-plussed,[7] I failed to add that this did not logically imply that similar processes happened systemically so that Cobb-Douglas applied, that a systemic theory of distribution did not have to match or reflect the processes at work at the level of the individual firm.

8. Kalecki's review of Keynes

The best illustration of my passed-over answer comes from Kalecki's remarkable 1936 review article of *The General Theory*, at that time untranslated from Polish.[8] In it, Kalecki starts with a profit-maximising, cost-minimising firm, the production technique of which could well be Cobb-Douglas, situated in either a purely (freely) competitive or an imperfectly competitive market. He nets out raw material costs and splits the value added implied by the net revenue and net cost curves into wage payments and surplus (=profits); he aggregates the values added of all firms in the economy to the economy as a whole and shows how wage-earners spending what they earn and profit-receivers receiving what they spend, given the level of investment expenditure, results in the overall levels of activity and employment, and the distribution of income between wages and profits, being determined simultaneously.

This two-sided relationship between accumulation and distribution was extended by Joan Robinson to the long period (in a Harrodian sense) in 1962 in her banana diagram (Joan Robinson (1962), 48, and even further by Donald Harris (1975, 1978) to take in the sphere of production in which the potential surplus is created as a result of the impact of the current state of the class war and the existing technical conditions of production. The realisation problem is analysed in the accompanying sphere of distribution and exchange in which the Keynesian "animal spirits" function and the Cambridge

[7] It was a toss up whether Bob's question or Paul correcting page proofs while I spoke was the more non-plussing.

[8] The first full translation was published in the December 1982 issue of *Australian Economic Papers*, see Targetti and Kinda Hass (1982).

saving function interact to determine the rate of accumulation and the distribution of income and so how much of the potential surplus is realised.

9. Concluding remarks

An essential part of setting up this alternative approach is Anwar's critique of Solow's methodology, of his theory and its application, and Anwar's recognition of the link between 'vision' and the specifics of theory, analysis and applied work. Mainstream analysis of firm's behaviour by no means implies that the system need mirror it. Anwar's contribution also puts paid to the late Charles Ferguson's, Ferguson (1969), and the late Mark Blaug's claims, Blaug (1974), that econometrics would decide how serious for neoclassical theory would be the results of the Cambridge – Cambridge capital theory controversies.[9] The hegemony and ignorance of the mainstream keeps this finding at bay but surely truth will ultimately prevail. If, when, it does Anwar's contributions will be major reasons why.

References

Bhaduri A. (1969), *On the significance of recent controversies on capital theory: A Marxian view*, The Economic Journal, 79(315), 532-539.

Blaug M. (1974), *The Cambridge revolution: success or failure? A critical analysis of Cambridge theories of value and distribution*, Institute of Economic Affairs, London.

Brown P. [Henry Phelps Brown] (1957), *The meaning of fitted Cobb-Douglas functions*, The Quarterly Journal of Economics, 71(4), 546-560.

Ferguson C.E. (1969), *The Neoclassical theory of production and distribution*, Cambridge University Press, Cambridge (UK).

Felipe J., McCombie J. S. L. (2013), *How sound are the foundations of*

[9] Joan Robinson further refuted the claim that econometrics could ride to the rescue by her repeated demonstration that comparisons of equilibrium positions (differences) cannot throw light on processes (changes), see, for example, Shaikh (115), n2.

the aggregate production function?, in Harcourt G. C. and Kriesler P. (eds.) (2013), *The Oxford Handbook of Post-Keynesian Economics*, Vol. 2, Critiques and Methodology, Oxford University Press, New York, 202-230.

Fisher F. M. (1971), *Aggregate production functions and the explanation of wages: a simulation experiment*, The Review of Economics and Statistics, 53(4), 305-325.

Foley D. K., Michl T. (1999), *Growth and distribution*, Harvard University Press, Cambridge (MA).

Harcourt G. C. (1969), *Some Cambridge controversies in the theory of capital*, Journal of Economic Literature, 7(2), 369-405.

Harcourt G. C. (1972), *Some Cambridge controversies in the theory of capital*, Cambridge University Press, Cambridge (UK).

Harcourt G. C. (1976), *The Cambridge controversies: old ways and new horizons - or dead end?*, Oxford Economic Papers, 28(1), 25-65.

Harcourt G. C., Peter Kriesler (eds) (2013), *The Oxford handbook of post-Keynesian economics*, Vol. 2, Critiques and Methodology, Oxford University Press, New York.

Harris D. J. (1975), *The theory of economic growth: a critique and reformulation*, American Economic Review, Papers and Proceedings, 65(2), 329-337.

Harris D. J. (1978), *Capital accumulation and income distribution*, Stanford University Press, Stanford.

Harrod R. F. (1939), *An essay in dynamic theory*, The Economic Journal, 49(193), 14-33.

Kaldor N. (1957), *A model of economic growth*, The Economic Journal, 67(268), 591-624.

Kalecki M. (1936), *Pare uwag o teorii Keynesa*, Ekonomista, 36(3) 18-26, En. tr. *Some remarks on Keynes' theory*, in Kalecki M. (1990), Osiatynksi J. (ed.), *Collected works in Michael Kalecki*, Vol. I, Capitalism, business cycles and full employment, Clarendon Press, Oxford, 223-232.

Nell E. J. (ed.) (1980), *Growth, profits and property. Essays in the revival of political economy*, Cambridge University Press, Cambridge (UK).

Robinson J. (1962), *Essays in the theory of economic growth*, Macmillan, London.

Robinson J. (1971), *Economic heresies: some old-fashioned questions in economic theory*, Macmillan, London.

Samuelson P. A. (1997), *Wherein do the European and American models differ?*, Address delivered at the Bank of Italy, October 2, 1997, No. 30, mimeo.

Shaikh A. (1973), *Theories of value and theories of distribution*, Ph.D. dissertation, Columbia University, New York.

Shaikh A. (1974), *Laws of production and laws of algebra: the Humbug production function*, The Review of Economics and Statistics, 56(1), 115-120.

Shaikh A. (1980), *Laws of production and laws of algebra: Humbug II*, in Nell E. J. (ed.) (1980), *Growth, profits and property*, Edward J. Nell (ed.), Cambridge University Press, Cambridge (UK), 80-95.

Shaikh A. (1997), *Marx's theory of value and the transformation problem*, in Schwartz J. (ed.), *The subtle anatomy of capitalism*, Goodyear Publishing Company, Santa Monica, 106-139.

Shaikh A. (1998), *The empirical strength of the labor theory of value*, in Bellofiore R. (ed.), *Marxian economics: a reappraisal*, Vol. 2, Macmillan, London, 225-251.

Shaikh A., Tonak E. A. (1994), *Measuring the wealth of nations: the political economy of national accounts*, Cambridge University Press, Cambridge (UK).

Simon H., Levy F. (1963), *A note on the Cobb-Douglas function*, The Review of Economic Studies, 30(2), 93-94.

Solow R. (1956), *A contribution to the theory of economic growth*, The Quarterly Journal of Economics, 70(1), 65-94.

Solow R. (1957), *Technical change and the aggregate production function*, The Review of Economics and Statistics, 39(3), 312-320.

Solow R. (1974), *Laws of production and laws of algebra: a comment*, The Review of Economics and Statistics, 56(1), 121.

Sraffa P. (1926), *The laws of returns under competitive conditions*, The Economic Journal, 36(144), 535-550.

Targetti F., Kinda-Hass B. (1982), *Kalecki's review of Keynes' General Theory*, Australian Economic Papers, 21(39), 244-260.

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BEYOND THE SURROGATE PRODUCTION FUNCTION

Abstract

In the early 1970s, two distinct and devastating critiques were specifically moved to the neoclassical production function. The first one was addressed to Paul Samuelson's 1962 Surrogate Production Function (SPF) by Garegnani (1970) on purely theoretical grounds, whereas the second radical critique was moved by Shaikh (1974) to Solow's 1957 attempt at giving support to the neoclassical theory of income distribution.

The paper will first go briefly through the works of different generations of neoclassical economists, by emphasising their respective different attitudes towards the neoclassical theory, in general, and the production function in particular. Some critiques of the neoclassical theory have been 'sterilised' over the years by the second generation of the neoclassical economists, whereas the Sraffian critique has been either patently eluded or, more simply, ignored altogether. Witness the reaction of Samuelson (1962) and especially the unseemly reaction by Solow (1974) to Shaikh (1974), on which the paper will provide some reflections. In particular, starting from Shaikh's suggestive insights, the paper will attempt at interpreting that very neoclassical attitude in general, by arguing that the production function looks like a Trojan horse, carrying with it a well defined representation and vision of the economy, which can be contrasted only by using and enriching more extensively the alternative conceptual framework laid down by Sraffa (1960).

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1. Introduction

An apparently naïve question might be asked quite repeatedly over the last 20 years or so: why the neoclassical paradigm is still so pervasive and dominant in economic theory and policy after Sraffa (1960), that is, after it has been found so irremediably defective on the logical grounds?

One of the consequences of this otherwise inexplicable situation is the stubborn insistence on the widespread use of the production function of the traditional neoclassical type in economic theory in general, and in most macroeconomic models in particular – notwithstanding Garegnani's 1970 early critique of the Samuelsonian 1962 Surrogate Production Function (SPF) and Shaikh's repeated critique (1974), (1980), (1987), which, through his Humbug production function, rightly ridiculed the neoclassical production function as being a meaningful analytical tool for the economic analysis.

One might reasonably suspect that *something else* lies beneath this very resistance, unlike what it would have happened in analogous circumstances in any other discipline proclaiming itself a 'scientific' discipline. Searching out this *something else* suggests quite naturally to start from the very beginning of the construction of the neoclassical paradigm, in the hope of finding some hints for answering the above question.

To begin with, it would be convenient to take as a reference two crucial dates: 1932 and 1962. The first one being the publication year of Robbins' well known methodological essay, whereas the second – as recalled before – being the publication year of Samuelson's SPF.

Each one of those dates divides quite sharply not only *different generations* of neoclassical economists, but represents also crucial turning points of *meaningful changes* undertaken by the neoclassical approach. Obviously enough, these two broad effects go hand in hand. It seems more convenient, however, to take them analytically apart.

It will be sustained that a first substantial change took place in the neoclassical economics since the publication of Robbins' 1932 methodological essay, which codified and generalise the Paretian 'philosophy' of a value-free, objective-based economic discipline. And, a far more second radical change, as compared with the works of the earlier neoclassical economists, did take place since the publication of Samuelson's SPF.

The paper will then go briefly through the works of some neoclassical economists of the 'first generation' (particularly Wicksteed and Wicksell) in order to emphasise their genuine effort, from their own perspective, in laying down the foundations of an alternative paradigm to the then prevailing classical paradigm. In their works, there can be found explicit recognition of some logical difficulties within the conceptual framework of the neoclassical theory – difficulties which have been completely ignored by the 'second generation' of the neoclassical economists from the early 1930s onwards and at least until the early 1960s. Over this period, in fact, this 'second generation' has been mainly occupied in refining and perfecting the Walrasian theory on the one hand, whereas, on the other, it has been occupied in defending the neoclassical paradigm from the Keynesian attack. However, whereas the Keynesian critique was in a sense quite easily to 'sterilise' and to downgrade to the status of 'special' and thus innocuous case *within* the traditional theory – a facilitated task due to the common reference framework and to the conceptual and analytical tools utilised all over the dispute – the Sraffian critique was instead logically impossible to oppose; it could only be possible either to elude it or, more simply, to ignore it altogether. Witness the reaction of Samuelson (1962) and especially the unseemly reaction of Solow (1974) to Shaikh (1974), on both of which the paper will provide some reflections.

The paper will then attempt at interpreting this very neoclassical attitude, by arguing that the production function looks like a Trojan horse, carrying with it a well defined representation and vision of the economy. A suggestion will be made in the direction of using and enriching more extensively the alternative conceptual framework laid down by Sraffa, in which every element characterising it has nothing whatsoever to do with the absolutely ruined and useless neoclassical one.

Generally speaking, it could be said that the one of the most characterising feature separating the two generations of neoclassical economists is reflected in their respective different *political* attitude in framing their own conception of the economy, and in the use they made of the analytical tools thereby employed. In particular, it will be emphasised that the *apology* of the market economy is a feature which strongly characterises the approach of the second and third generations of neoclassical economists. It will be maintained, in fact, that the analytical framework employed in the representation of the economy is intrinsically neither political-neutral nor value-judgement-free, for it conveys a well defined *political* conception.

This might sound perhaps too obvious, had not the debates, which took place after Sraffa (1960), generally eluded this very point, thus failing in bringing it properly to the fore. The critique of the neoclassical theory, in fact, has generally been made on logical grounds, and the Sraffa framework has been mainly used for the support it gives to the critique of the notion of 'capital', rather than for the new perspective it opens and suggests. From this very viewpoint, Shaikh's contribution appear to be fundamental, for it does open a new perspective for the critique of neoclassical theory, by soliciting the unravelling of its ideological and apologetic character.

2. The foundational period (1871-1932)

In the last quarter of the 19th century the neoclassical economists strongly believed of having inherited 'defective', not amendable theories from the classical economists and from Marx. Hence the need of a new conceptual framework.

However, from the very beginning there already existed problems concerning the formulation of the production function and the interpretation of the income distribution thereby implied. Witness Wicksteed's (1894) 'adding up problem' and Wicksell's (1901) notion of 'capital'.

At the basis of the 'adding up problem' there was the need of singling out a *common criterion* (possibly 'objective' and 'equitable', at the same time) simultaneously applicable to each and every 'factor' of production for the determination of its own remuneration. Moreover, this common criterion was strictly linked with the 'marginal productivity' of each and every 'factor' of production. By framing the problem of income distribution in that way would have had the immediate consequence for the whole product of the economy of being completely 'exhausted', without leaving, therefore, any kind of 'residue' after its distribution had been accomplished.

This new framework does appear in sharp contrast to the classical theory, where the notion of a 'surplus' was playing instead a crucial role in the division of the social product and where any share of income was receiving its own determination through a criterion established from outside the system of production. It should also be noted that one of the consequences of the 'adding up problem' was that of putting automatically every kind of 'factor' (both original, like labour and land, as well as produced, like capital goods) on the same footing. An important exception, in this regard, was Wicksell (1901) – as will be seen presently.

The economic conditions of the 'adding up problem' are very well known: perfect competition and constant return to scale, which Flux's (1894) prompt review to Wicksteed (1894), rigorously translated into analytical terms by applying a well known Euler's theorem in economics for the first time.

However, Wicksteed's analytical effort of systematically framing the 'adding up problem' was quite heavily criticised by neoclassical economists of the time, like Walras (1894), Pareto (1897) and Edgeworth (1904).

Walras was extremely critical of the marginal productivity theory as stated by Wicksteed. In fact, he added a postscript (dated 1895) in the Appendix III (dated 1894) which appeared in the 3rd edition of his *Eléments* (1896) only. In that postscript, he blames Wicksteed for *not* having extended the marginal productivity theory to non-homogeneous and to non-linear functions. He makes it also clear that the marginal productivity theory, important as it is, is however 'not relevant to the determination of the prices of the services', p. 495. This is essentially what is contained also in Lesson 31 of his *Eléments*, as Walras explicitly states, blaming again with unusual harshness Wicksteed for the fact that he 'fell short of establishing it for the more general case and would have been better inspired if he had not made such efforts to appear ignorant of the works of his predecessors', *ibidem*. Walras, however, seems to be uncomfortable with the whole issue here under consideration, if one considers the various changes he introduced in Lesson 36 of his *Eléments* – not to mention the new heading given to that chapter 'The Marginal Productivity Theory', judged arguably 'significant' by Jaffé in a special note to the English edition of the *Eléments*, p. 604.

Not less critical than Walras was Pareto (1897) of the marginal productivity theory in general and of Wicksteed's 'adding up problem' in particular. He objected to the assumption of constant returns to scale in terms of physical product and was also critical about the validity of the marginal productivity theory 'without corrections', *ibidem*, p. 83.

Edgeworth (1904) and (1925) must also be included in the list of those critics who were never convinced by the theory of income distribution as stated by Wicksteed. He never accepted, for instance, the fact that the entrepreneurs – under the stated conditions – would have received no reward for their works. He was even quite sarcastic with Wicksteed, to the point of showing some kind of 'scientific' disrespect for him – as the following passages testify:

> In fact, he [Wicksteed] finds that the product depends upon the factors by a relation which mathematicians designate a "homogeneous function of the first degree". This is a remarkable discovery; for the relation between product and factor is to be considered to hold good irrespectively of the play of the market […]
>
> There is a magnificence in this generalisation which recalls the youth of philosophy. Justice is a perfect cube, said the ancient sage; and rational conduct is a homogeneous function, add the modern savant. A theory which points to conclusions so paradoxical ought surely to be enunciated with caution. Edgeworth (1904), p. 31.

A lively debate took place in the 1930s between Hicks (1932a), (1932b), Schultz (1932) and Joan Robinson (1934). Steedman (1987) reconstructs and summarises very neatly the analytical debates surrounding the whole 'adding up problem', emphasising, in particular, the circumstance according to which Wicksteed, in accepting the criticism raised by Pareto and Edgeworth, would have retreated the *extension* of his theorem to the case of imperfect markets, but *not* its original solution, which was believed still valid in the case of perfectly competitive markets. However, Steedman seems to have overlooked an important letter written by Wicksteed in 1916 to J. M. Clark, in which he declares to have definitely abandoned the whole theory:

I was and am extremely interested in your independent arrival at my old conclusion; and it would be interesting indeed if you were to rehabilitate it after I had abandoned it. *But I fear it cannot be done*. What upset me was a remark of Pareto's […] [He] said that the variables not being independent vitiated the argument. Letter by Wicksteed to J. M. Clark, dated February 14th, 1916, quoted in Dorfman (1964), italics added.

Before considering Wicksell's original contribution on the specific topic so far discussed, it would be of some interest to briefly take into account some important reflections by Paul Samuelson appearing in his well known *Foundations of Economic Analysis* (1947). In this work several pages are dedicated to the 'adding up problem', and Samuelson's effort seems to consist in rendering the marginal theory of distribution *compatible* with any form whatsoever of returns to scale within the wider neoclassical theory of competitive equilibrium. In this respect, the basic philosophy underlined the maximization principle (the equality between price and marginal cost at firm level) is considered sufficient and perfectly in line with the principles characterising the marginal productivity theory, being the ultimate objective of the entrepreneur the maximising of 'profit', and not the minimising of the average cost. Hence, the sole equality between price and marginal cost is being retained just compatible with the equality between the price of a 'factor' and its value marginal productivity. Viewed form this angle, the 'adding up problem' – according to Samuelson – would be a 'no problem', even by considering the case of decreasing returns to scale (and therefore even if a 'residue' exists in favour of the entrepreneur).

Samuelson's contribution, however, goes still further, as he tries to re-define in a more 'appropriate' way what until then were generically called 'factors of production' – putting aside, once more, Wicksell's special contribution.

Samuelson writes:

> It is useful, I believe, to avoid the expression "factor of production" entirely. This has been used in at least two senses, neither of which is quite satisfactory. First, it has been used to denote broad composite quantities such as "labor, land, and capital". On the other hand, it has been used to denote any aspect of the environment which has any influence on production. I suggest that only "input" be explicitly included in the production function, and that this term be confined to denote *measurable quantitative economic goods or services*.", Samuelson (1947), p. 84, italics added.

The last sentence should be connected, in a sense, with what years later he named 'surrogate production function'. For the moment, however, it could be conveniently taken as a good starting point for better evaluating Wicksell's contribution, which – emphatically but not redundantly – has been repeatedly referred to above.

Wicksell can certainly be considered an 'heretic' neoclassical economist, and not only for the topic here under discussion.

There are at least three aspects of his constructive contribution which deserve attention here:

(i) the analytical solution he provided to the 'adding up problem';
(ii) the definition he gave of, and the treatment he reserved to, the notion of 'capital';
(iii) the logical implication to be drawn from his theoretical model of a stationary economy.

As to the first aspect, Wicksell carefully and neatly distinguished *non*-capitalistic production from capitalistic production, the former being characterised by the presence of *non*-produced input only (labour and land), whereas the latter contemplates also produced input (capital goods). The 'adding problem' is rigorously stated by Wicksell only within the first of the two above frameworks, not least for the reason that the arguments of a 'production function', *i.e.* the input of production, can legitimately be conceived and analytically be inserted in it, as they are measurable in technical units independent of prices, and the notion of 'marginal productivity' as referred to each one of them can be conceived as well.

In Wicksell's own words:

With labour and land, as we have already pointed out, the law of marginal productivity applies, with certain reservations, both to the economy as a whole and to every private undertaking. […] But this theory applies only to capital, as usually conceived, when we look at it from the point of view of the individual entrepreneur, to whom wages and rent are data, determined by the market. If we consider an increase (or perhaps a decrease) in the total capital of society, then it is by no means true that the consequent increase (or decrease) in the total social product would regulate the rate of interest. Wicksell (1901), p. 148.

Whereas labour and land are measured each in terms of its own technical unit […] capital, on the other hand, […] is reckoned, in a common parlance, as a sum of exchange value" – whether in money or as an average of products. […] it is a theoretical anomaly which disturbs the correspondence which would otherwise exist between all the factors of production. *Ibidem*, p. 149.

If capital also were to be measured in technical units, the defects would be remedied and the correspondence would be complete. But, in that case, productive capital would have to be distributed into as many categories as there are kinds of tools, machinery, and materials, etc., and a unified treatment of the role of capital in production would be impossible. *Ibidem*.

It is within the context referred to above that Wicksell introduced for the first time in economics a particular production function which some decades later became known as the Cobb-Douglas (1928) production function. Making use of such a function and of its analytical properties, Wicksell was so able to uncover the basic features as well as the limits of an income distribution based on the principle of the marginal productivities of the 'factors'.

The definition he gave of, and the treatment he reserved to, the notion of 'capital' were perfectly coherent with his framing the problem of income distribution and with the abysmal difference he so clearly put to the fore between produced and *non*-produced input. As a consequence, he directed his criticism against von Thünen, for his having put on the same footing wages and interest as being determined on the same symmetrical principle of 'marginal productivity', *ibidem*, p. 147, but also against Walras, for his dangerous arguing in a circle, in so far as capital goods were not treated with extreme precaution:

Again, it is futile to attempt – with Walras and his followers – to derive the value of capital-goods from their own cost of production or reproduction; for in fact these costs of production include capital and interest, whereas our analysis of the laws of the cost of production has hitherto proceed on the assumption that production is non-capitalistic. *We should, therefore, be arguing in a circle*. *Ibidem*, 149, italics added.

Wicksell tried to escape from all the intricacies which surrounded the notion of 'capital', by constructing a model with dated quantities of non-produced inputs as 'representatives' of capital goods.

The simplest model one might conceive in this respect is that representing a stationary economy with only one single final product and dated quantities of labour and land over n periods, and composed of $2n + 3$ independent equations: 1 production function (whose arguments are the dated quantities of labour and land over the n periods), $2n$ 'marginal productivity' conditions, and 2 demand and supply equality conditions for labour and land respectively.

The above system of equations does have $2n + 4$ unknowns: the quantity of the final product, the $2n$ dated quantities of labour ad land, the wage rate, the rent rate and the rate of interest (rate of profit). If the system is solvable, one of the unknowns can thus be fixed from outside the system of production, and – obviously enough – the most eligible unknown which might be fixed in this way is just the rate of interest. The latter possibility straightforwardly implies the unavoidable circumstance that distribution cannot find a consistent solution from within the model of production and also that 'capital' – *however defined* – cannot be measured independently of distribution. Moreover, the notion of 'marginal productivity' of any 'factor', although formally definable, turns out to be absolutely irrelevant in relation to the distribution of income.

As far as the above model is concerned, it is therefore necessary to sharply distinguish the formal solution as given by Wicksell in his *Lectures*, and consisting in fixing the *value* of capital, from the alternative possible solution to that same model and consisting – as we have seen – in fixing the rate of interest from outside the system of production – a solution not so dissimilar from that given by Sraffa (1960), p. 33, to his system of equations.

3. Robbins' 1932 essay and the consolidation period (1932-1959)

Robbins' 1932 essay can be considered as a dividing line between two profoundly different periods, as far as the development of the economic ideas is concerned.

As has been seen above, in fact, neoclassical economics, though already possessing a well structured framework of reference, was not able to gather, until Robbins' work was published, a unanimous consensus on crucial parts of its theoretical basic construction and, what was more, it was still missing an explicit and organic methodological program which might possibly unify the different models within a well ordered 'paradigm'. To this task Robbins' work is fundamentally addressed. Its features, by taking advantage of the Pareto own approach in economics, pointed to the direction of freeing economics from any value judgement and ethical consideration, by means of which economics could be aligned with any other 'hard science', like Mathematics, and gaining in this way the status of an objective-based discipline. As Robbins (1932) explicitly writes 'Economics is entirely neutral between ends', p. 16, '[economics] is in no way to be conceived, as we conceive Ethics and Aesthetics, as being concerned with ends as such', p. 32, '[economics] cannot pronounce on the validity of ultimate judgment of value', p. 147.

In addition, the contrast with the classical political economy is far more evident, for the very essence of economics, according to Robbins, lies in stating criteria which help making the best possible choices, 'objectively' and 'individually' made, in a world constrained by scarce resources. The classical notion of the 'Social Product' becomes a meaningless notion and production and income distribution come to be simultaneously determined from within the same analytical framework. As Robbins put it:

> We no longer enquire concerning the causes determining variations of production and distribution. We enquire rather concerning the condition of equilibrium of various economic 'quantities' […] Instead of dividing our central body of analysis into a theory of production and a theory of distribution, we have a theory of equilibrium […] Instead of regarding the economic system as a gigantic machine for turning out an aggregate product and proceeding to enquire what

causes make this product greater or less, and in what proportions this product is divided, we regard it as series of interdependent but conceptually discrete relationships between men and economic goods. *Ibidem*, pp. 67-68.

Lots of valuable works were published over a quarter of century since Robbins' 1932 essay, whose unifying thread might be found in a work of refinement and consolidation of the neoclassical paradigm in a more compact and solid form, with respect to works of the neoclassical economists of the first generation. At the very beginning, this process was mainly analytical, and so the aim to be pursued was explicitly that of refining or amending, according to circumstances, previous neoclassical works – witness the works by Neisser (1932), Zeuthen (1932), von Stackelberg (1933) and Wald (1933-34), (1934-35).

Soon after the publication of Keynes' 1936 *General Theory*, however, the works by Hicks (1939), Samuelson (1947), Arrow-Debreu (1954), McKenzie (1954), Gale (1955), Nikaidô (1956), Debreu (1959) should instead be read and interpreted in a different light. Hicks' 1939 *Value and Capital* should also be mentioned in this connection, for the far strongest, pervasive and long lasting impact it had in the economic profession. However, it was Hicks himself that many years later recalled a visit he made to the United States and a meeting he had with some economists as Samuelson, Arrow, Milton Friedman, and Don Patinkin. He writes:

I did not know them, but they knew me; for I was the author of *Value and Capital*, which (as since become obvious) was deeply influencing their work. They regarded it as the beginning of *their* 'neo-classical synthesis' – no more than the beginning for they and their contemporaries, with far more skill in mathematics than mine, were sharpening the analysis I had merely roughed out. But I am afraid I disappointed them, and I have continued to disappoint them. The achievement have been great; but *they are not in my line. I have felt little sympathy with the theory for theory's sake, which have been characteristic of one strand in American economics; nor with the idealisation of the free market, which has been the characteristic of another*. Hicks (1983), p. 361, italics added, except the first two.

In the 1950s another process was also taking place. It basically consisted in annihilating the 'revolutionary' impact of Keynes' *General Theory* by ultimately assimilating its propositions within the paradigm of neoclassical economics. The *unconstructive* mathematical proofs of 'existence' of an equilibrium[1] served, on the one hand, in reinforcing the structure and the inner message of the overall beneficial effects of the 'invisible hand' of the market; on the other, it gave also much analytical and philosophical support to the so-called 'neoclassical synthesis' (and also to the other subsequent 'syntheses' which followed afterwards) to the effect of producing a *very strong defence* of the 'market' as the pivotal institution regulating every kind of relationship among individuals. This is one of the *new key features* neoclassical economics began to possess and to subtly insert into its own 'research program' in the years to come – in contrast to the attitude which instead was characteristic of the neoclassical economists of the 'first generation', as has been seen above.

4. Towards Sraffa (1960)

Meanwhile in the early 1950s, and until the Sraffa book was published (1960), Sraffa's (1951) remarkable *Introduction* to Ricardo's *Principles* came out. It can be read as the most appropriate 'prelude' to his subsequent 'prelude to a critique of economic theory', for he provided an assessment of the classical theory of value and also discretely paved the way to the rehabilitation of classical political economy – as subsequently it became evident with his 1960 book.

In the first instance, Sraffa's 1951 *Introduction* to Ricardo should be acknowledged as an extraordinary 'editorial' event – for it is ultimately the end result of the magnificent long waited editing work by Sraffa himself on Ricardo's own works and correspondence. At the same time, however, it is very much worth noticing and emphasising that that event was taking place in a specific historical period in which much effort was being devoted to neutralise Keynes' theoretical attack to the economic orthodoxy and in which McCarthyism was bravely made its victims, even within the economic profession.

In Cambridge (England), over that same period, an increasing concern was coming into being as regards problems of *logical consistency* of neoclassical theory.

[1] On this specific point cf. Velupillai (2002), (2011).

Joan Robinson's attack to the production function (1953) is perhaps one of the most popular and significant example, although, it should honestly be noted, her work seems to be not so original after Wicksell's criticism of the notion of 'capital' employed by the neoclassical theory.

A very less known criticism – albeit far more profound than Joan Robinson's and with much more 'Sraffian flavor' – was Graaff's firm rebuttal of the conceptual separation between 'size' and 'distribution' of income.

> In a one commodity world – Graaff writes – some definite meaning could be attached to a phrase like 'the size of the national income' […] But as soon as we leave a one commodity world this ceases to be true. There is no unambiguous meaning which we can attach to 'the size of the national income' when we have a heterogeneous collection of goods and services. […] [A Paretian welfare function] will only tell us what weights to use when the distribution of the goods among the members of the community is given. […] [T]he size-distribution dichotomy is inconsistent with the basic Paretian value judgments that individual preferences are to count and that a *cet. par.* increase in any one man's well-being increases social well-being. […] Moreover, 'size' in this sense will generally change whenever the distribution change, even if the collection of goods and services distributed remains the same. Graaff (1957), pp. 91-92.

Graaff's punctual criticism on a vital part of neoclassical economics came out almost unnoticed in the literature, perhaps because it made its appearance in a 'wrong' book. His criticism, however, went up to the point of proving the *logical impossibility* of conceiving a market for saving and for investment. In so doing, he went far beyond the Keynesian intriguing world of 'uncertainty', and perhaps far more deeply than Joan Robinson's 'circular reasoning'.

In the middle 1950s, Solow (1956) and (1957) was laying down the foundations of a macroeconomic neoclassical theory of growth, in an academic environment in which Keynes was still under attack, with the first neoclassical synthesis almost accomplished, and the economic theory monopolized by the recently published Arrow-Debreu model (1954) of General Equilibrium.

A side but important effect of Solow's macroeconomic contribution in the 1950s was that of conveying the underlined 'philosophy' of the neoclassical economics more straightforwardly and more compactly than the Walrasian models *à la* Arrow-Debreu, through the emphasis given to factor substitution in the equilibrium process and to the distribution of income based on the marginal productivities of the 'factors'.

In 1959, Pasinetti wrote a long comment on Solow (1957) article on technical change and the production function. Starting from the crucial distinction between labour and land, on the one hand, and capital, as produced means of production, on the other – a distinction first brought about clearly by Wicksell, as has been seen earlier in this paper – he emphasises the profound different nature of 'capital', and the consequent necessity of introducing a parallel production function for the capital goods industry. In this way, he obtains results in opposite direction than Solow's. Solow's (1959) candid and firm reply to Pasinetti is much worth mentioning, by noting both the form and the substance of his own comment.

The *form* runs as follow:

> Mr. Pasinetti *carefully thought-out article* gives clear expression to one important view of production and technological change. I enjoyed reading it, but I must admit that *I disagree with much of it*. Solow (1959), p. 282, italics added.

As to the *substance* of Solow's reply, it goes as follows:

> A notion like "the stock of capital" is no problem for the theory [of General Equilibrium] because the notion never occurs; "real capital" is not brought and sold on any market. […]
>
> I had in mind the simplest of all capital models, what might be called the Ramsey model, in which the same commodity or composite commodity serves both as consumption good and as capital good (wheat which can be eaten or used as seed). *Ibidem*, p. 284.

Pasinetti, in his reply to Solow, acknowledged only his '*calm and carefully* framed criticism'. Pasinetti (1959), p. 285, italics added. However, he did not emphasise Solow's apparent 'schizophrenia', in his asserting that 'the "stock of capital" is no problem' for the *disaggregate* model of General Equilibrium, while his *aggregate* model had the same commodity serving 'both as consumption good and as capital good'.

It should also be mentioned at this juncture that just in the year of publication of Sraffa's book Garegnani was also publishing the Italian version of his 1959 PhD Cambridge thesis, which basically consists of a long analytical journey through the main theories of income distribution, aiming at bringing about some *logical* difficulties underneath the notion of 'capital'.

5. Sraffa (1960) and the reaction period (1962-1974)

The publication of Sraffa's *Production of Commodities by Means of Commodities* (1960) induced neoclassical economics to a radical change towards its own critics. The logical basis of its entire 'paradigm' was felt to be dangerously at peril.

Paul Samuelson (1962) promptly 'invented' a Surrogate Production Function (SPF) as a means of providing a *rationalization* of the J. B. Clark parables and also a *justification* for the use of aggregates. More subtly, however, the SPF should be seen as a strong and bold reaction to Sraffa's critique of the notion of 'capital' in economic theory.

More generally, had the SPF been proved all right, then the main features of neoclassical economics would have been validated as well – Sraffa's *alternative* paradigm notwithstanding.

Garegnani (1970) definitely proved the *non generality* of the SPF, *i.e.* if different commodities are produced then no 'aggregate production function' exists.

However, if the 'aggregate production function' was 'killed', it was not yet 'buried', as it continued to survive all around in the economic literature.

In effect, over the years following the publication of Sraffa's book there took place a lively debate centred on *capital theory* and on problems pertaining the *logical consistency* of neoclassical theory. Both of them are vital parts of the whole neoclassical paradigm, but nevertheless only *parts*. Macroeconomics was undertaking a path safely leading to a pre-Keynesian world and the pernicious Sraffian effects of the capital theory controversy were very soon jettisoned and completely forgotten.

Much support to neoclassical economics was still given by some empirical data, which seemed to be fitted pretty well by some standard production functions, the outstanding of all being the celebrated (Wicksell) Cobb-Douglas production function.

6. Shaikh's refined contribution

In an extraordinarily dense and punctual essay, Shaikh (1974) destroys with chirurgical analytical fineness the empirical basis for aggregate neoclassical production functions.[2]

He starts considering the standard aggregate data of an economy, by separating the value of output, wages and profits, and the index numbers for capital and labour. By working out on a series of *identities*, which can easily be derived from those data, he quite naturally arrives at an equation which is simply an *algebraic relationship*, identical to a linearly homogeneous Cobb-Douglas production function, with neutral technical progress and satisfying marginal productivity 'factor rewards'. To get this result, it suffices considering *any* output-input data, coming out even from *implausible* economies – such as the fantastic HUMBUG economy – so long as shares are being constant. As a consequence, the algebraic relationship which is the end result of a simple mathematical manipulation, cannot be interpreted as a true production relationship, and therefore the *mathematical* function cannot seriously be taken as a *production* function.

Shaikh applies this strong analytical result in different ways to substantiate his own thesis. In doing this, he easily finds Solow's attempt at measuring the contribution of technical change to economic growth a fictitious and useless exercise, being actually Solow's 'production' function a simple 'mathematical' function derived from an identity and so holding true for any production and distribution behaviour. Shaikh's critique seems thus to have definitely destroyed the last possibility for the survival of the neoclassical theory of distribution.

The one-page annoyed (and puzzling) comment by Solow (1974) seems only to reflect theoretical uneasiness and heavy embarrassment. It should also be noted that Solow once remarked:

[2] It is worth mentioning here Zambelli (2004), in which the difficulty of representing production by means of an aggregate neoclassical production function is made by use of computer simulations.

[T]he marginal productivity theory is fundamentally a micro-economic theory and its presentation in macro-economic terms is an act of *empirical desperation*. Solow (1968), p. 449, italics added.

Shaikh's original contribution leads straightforward to remarkable results. Firstly, putting aside the *theoretical* weakness of neoclassical theory, the much emphasised *empirical* support by most neoclassical economists to one of its supposedly strong analytical tool, such as the Cobb-Douglas production function, completely collapses as well, with the consequence that all the results which are supposed to be produced by a competitive market collapse too.

Secondly, in finely separating out what he calls the *laws of algebra* from the *laws of production*, he makes it clear that still there was *further analytical work* to be done for accomplishing the critique of neoclassical theory – further analytical work, with respect to that already done until then by the capital theory controversy of the 1960s on a purely theoretical ground.

Thirdly, Shaikh's contribution seems ultimately to give much emphasis to the *non-neutrality* of the analytical tools used in economics and to attribute great importance to the *relevance* of an economic theory. On both sides, neoclassical economics appears to be a very feeble and poor theoretical construction.

All this opens the way to further reflection, in the direction of pursuing *alternative* reference frameworks for representing the economy. Suffices it here to recall that Shaikh (1987) rightly saw 'capital' not only as a mere means of production, but also as *social relation* and that he wrote not only on the *laws of algebra* but also on the *wealth of algebra vs* the poverty of theory (1982) – as referring to the unnecessary mathematical 'reframing' of the Sraffian theory by the so-called Neo-Ricardians, leading to an inevitable loss of *relevance* of its still fruitful classical and Marxian roots.

References

Arrow K. J., Debreu G. (1954), *Existence of an equilibrium for a competitive economy*, Econometrica, 22(3), 265-290.

Cobb C. W., Douglas P. H. (1928), *A theory of production*, American Economic Review, 18(1), 139-165.

Debreu G. (1959), *Theory of value. An axiomatic analysis of economic equilibrium*, Yale University Press, New Haven and London.

Dorfman J. (1964), *Wicksteed recantation of the marginal productivity theory*, Economica, 31, 294-295.

Eatwell J., Milgate M., Newman P. (eds.) (1987), *The new Palgrave: a dictionary of economic theory and doctrine*, The Macmillan Press, Lts., London.

Edgeworth F. Y. (1904), *The theory of distribution*, The Quarterly Journal of Economics, 18(2), 159-219.

Edgeworth F. Y. (1925), Review of Clark J. M. (1923), The Economic Journal, 35, 245-251.

Flux A. W. (1894), *Review of Philip H. Wicksteed's essay on the coordination of the laws of distribution*, The Economic Journal, 4, 308-313.

Gale D. (1955), *The law of supply and demand*, Mathematica Scandinavia, 4, 143-146.

Garegnani P. (1960), *Il capitale nelle teorie della distribuzione*, Giuffrè, Milano.

Garegnani P. (1970), *Heterogeneous capital, the production function and the theory of distribution*, The Review of Economic Studies, 37(3), 407-436.

Graaff J. de V. (1957), *Theoretical welfare economics*, Reprint (1967), Cambridge University Press, London.

Hicks J. R. (1932a), *Marginal productivity and the principle of variation*, Economica, 12, 79-88.

Hicks J. R. (1932b), *A rejoinder, [Marginal productivity and the Lausanne School, a reply to Henry Schultz]*, Economica, 12, 297-300.

Hicks J. R. (1939), *Value and capital. An inquiry into some fundamental principles of economic theory*, Oxford University Press, London, 2nd ed. (1946).

Hicks J. R. (1983), *The formation of an economist*, in Banca Nazionale del Lavoro Quarterly Review, 130, 195-204.

Hicks J. R. (1983), *Classics and moderns*, Basil Blackwell, Oxford.

Keynes J. M. (1936), *The general theory of employment, interest and money*, Macmillan and Co. Ltd, London.

McKenzie L. W. (1954), *On equilibrium in Graham's model of world trade and other competitive systems*, Econometrica, 22(2), 147-161.

Neisser H. (1932), *Lohnhöhe und Beschäftigungsgrad im marktgleichgewicht*, Weltwirtsch. Arch., 36, 413-455.

Nell E. J. (ed.) (1980), *Growth, profit, and property: essays in the revival of political economy*, Cambridge University Press, Cambridge (UK).

Nikaidô H. (1956), *On the classical multilateral exchange problem*, Metroeconomica, 8(2), 135-145.

Pareto V. (1897), *Cours d'économie politique*, Vol. II, F. Rouge, Lausanne.

Pasinetti L. L. (1959), *On concepts and measures of changes in productivity*, The Review of Economics and Statistics, 41(3), 270-282.

Pasinetti L. L. (1959), *Reply* [to Robert Solow] (1959), The Review of Economics and Statistics, 41(3), 285-286.

Robbins L. (1932), *An essay on the nature and significance of economic science*, Macmillan and Co., Ltd, London, 2nd ed. (1935).

Robinson J. (1934), *Euler's theorem and the problem of distribution*, The Economic Journal, 44(175), 398-141.

Robinson J. (1953), *The production function and the theory of capital*, The Review of Economic Studies, 21(2), 81-106.

Samuelson P. A. (1947), *Foundations of economic analysis*, Harvard University Press, Cambridge (MA), Massachusetts, enlarged ed. 1983.

Samuelson P. A. (1962), *Parable and realism in capital theory: the surrogate production function*, The Review of Economic Studies, 29(3), 193-206.

Schultz H. (1932), *Marginal productivity and the Lausanne School*, Economica, 37, 285-296.

Shaikh A. (1974), *Laws of production and laws of algebra: the Humbug production function*, The Review of Economics and Statistics, 56(1), 115-120.

Shaikh A. (1980), *Laws of production and laws of algebra: the Humbug II*, in Nell E. J. (ed.) (1980), *Growth, profits and property*, Cambridge University Press, Cambridge (UK), 80-95.

Shaikh A. (1982), *Neo-Ricardian economics. A wealth of algebra, a poverty of theory*, Review of Radical political Economics, 14(2), 67-83.

Shaikh A. (1987), *Capital as a social relation*, in Eatwell J., Milgate M., Newman P. (eds.) (1987), *The new Palgrave: a dictionary of economic theory and doctrine*, The Macmillan Press Lts., London, 333-336.

Solow R. (1956), *A contribution to the theory of economic growth*, The Quarterly Journal of Economics, 70(1), 65-94.

Solow R. (1959), *Comment* [on Pasinetti L.] (1959), The Review of Economics and Statistics, 41(3), 282-285.

Solow R. (1968), *Distribution in the long and short run*, in Marchal J. and Ducros B. (eds.), *The distribution of national income*, Proceedings of a Conference held by the International Economic Association at Palermo, Macmillan, NY, 449-475.

Solow R. (1974), *Laws of production and laws of algebra: the Humbug production function: a comment*, The Review of Economics and Statistics, 56(1), 121.

Sraffa P. (1951), *Introduction*, in Sraffa P. (ed.),*The works and correspondence of David Ricardo*, Vol. I, Cambridge University Press, Cambridge (UK), xiii-lxii.

Sraffa P. (1960), *Production of commodities by means of commodities. Prelude to a critique of economic theory*, Cambridge University Press, Cambridge (UK).

Velupillai K. (2002), *Effectivity and constructivity in economic theory*, Journal of Economic Behavior and Organization, 49, 307-325.

Velupillai K. (2011), *Freedom, anarchy and conformism in academic research*, Interdisciplinary Journal of Economics and Business Law, 1(1), 85-110.

von Stackelberg H. (1933), *Zwei kritische bemerkurgen zur preistheorie Gustav Cassel*, Zeitschrift für Nationalökonomie, 4, 456-472.

Wald A. (1933-34), *Über die eindeutige positive lösbarkeit der neuen produktionsgleichungen (I)*, Ergebnisse eines mathematischen Kolloquiums, Heft 6, Menger K. (ed.), Franz Deuticke, Leipzig und Wien, (1935), 12-18.

Wald A. (1934-35), *Über die produktionsgleichungen der ökonomischen wertlehre (II)*, Ergebnisse eines mathematischen Kolloquiums, Heft 7, Menger K. (ed.), Franz Deuticke, Leipzig und Wien, (1936), 1-6.

Walras L. (1874), *Éléments d'économie politique pure, Corbaz, Lausanne*, translated by William Jaffé, Edition Défi nitive 1926, George Alen and Unwin Ltd, London, 1954.

Wicksell K. (1901), *Föreläsningar i nationalekonomi, första delen: teoritisk nationalekonomi*, Lund. English edition: Lectures on Political Economy, tr. by Classen E. and edited with an Introduction by Lionel Robbins, Vol. I, Macmillan, New York.

Wicksteed P. H. (1894), *An essay on the co-ordination of the laws of distribution*, Macmillan, London.

Zambelli S. (2004), *The 40% neoclassical aggregate theory of production*, Cambridge Journal of Economics, 28(1), 99-120.

Zeuthen F. L. B. (1933), *Das prinzip der knappheit technische kombination und ökonomische qualität*, Zeitschrift für Nationalökonomie, 4, 1-24.

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UNCONVENTIONAL RESULTS WITH SURROGATE PRODUCTION FUNCTIONS

Abstract

Starting from Samuelson's famous 1962 Surrogate Production Function, we show that some such functions are perfectly consistent with a 'return of the machine type'. This is so when we introduce a distinctly conventional possibility—that of machine–labour substitution—instead of Samuelson's fixed proportions, within every technique. We also show that, in the presence of two primary inputs, the surrogate model can give distinctly unconventional primary input use/input price relations at the level of the consumer good industry, even if the rate of interest is identically null.

JEL Classification: C67, D24.

Keywords: P. A. Samuelson, Surrogate Production Function, Machine Types, Corn-Machine Model.

1. Introduction

In Paul Samuelson's famous Surrogate Production Function of 1962, the smallest change in the rate of interest provokes a change in the type of machine in use. He associated the different machines

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with 'separate production functions' (Samuelson, 1962, p. 194), but no substitution possibilities were allowed between the use of any *given* type of machine and the use of labour. Here we modify Samuelson's model precisely by allowing for such substitution, while retaining his assumption that any given type of machine can be used, with labour, either to produce new machines of the same kind or to produce the consumption commodity.

Samuelson seemed to take it for granted that the 'surrogate' properties should hold *a fortiori* if each production function was characterized by 'smooth substitutability and well behaved marginal productivity partialderivatives' (Samuelson, 1962, p. 200). As he put it, '*Even* in our discreteactivity fixed coefficient model of heterogeneous capital goods, the factor prices (wage and interest rates) *can still* be given various long-run marginalism (*i.e.* partial derivative) interpretations' (Samuelson, 1962; our italics). Yet we show below that substitution possibilities between each machine type and labour can make Samuelson's model *less* akin to the conventional neoclassical model than is the fixed-coefficients case.

In section 4 of the paper we allow for land use. A change in the rent/wage ratio, at a constant rate of interest, generally determines a switch from one machine type to another and we analyse the change in labour and land use in relation to the wage and rent rates. We show that, adopting the logic of Samuelson's surrogate model, there is no guarantee that both labour and land use (per unit of output) be inversely related to their own price, even if the rate of interest is identically null.

Needless to say, Samuelson was able to construct his Surrogate Production Function only because he allowed himself the very strong assumption of 'equal proportions', namely that, for each machine, the machine–labour ratio is the same whether new machines or the consumer good are being produced. When we now allow for variable machine–labour ratios, for each type of machine, the analogous (very strong!) assumption is that the unit cost functions for machine production and for consumer good production are proportional to each other. Indeed, by choice of measurement unit for the quantities of the machine, we may make the two unit cost functions the same.

In an obvious notation, then, with circulating capital

$$p = c[(1+i)\,p, w] \equiv \gamma[(1+i)\,p, w] = \pi$$

With $p = \pi = 1$ we can write the corresponding wage rate/interest rate frontier as

$$c[(1+i), w] = 1 \tag{1}$$

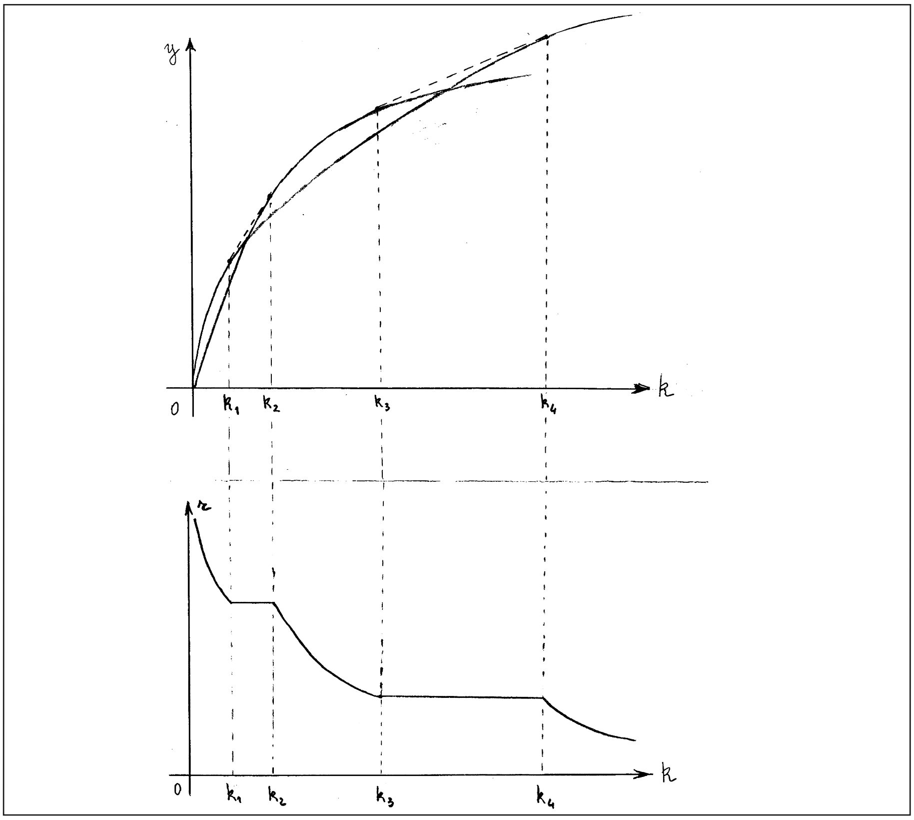
2. Two machine types

Let there be just two kinds of machine, to begin with, each yielding a *w(i)* frontier such as (1). In Samuelson's case all such frontiers are straight lines but now that we have allowed for machine–labour substitution any such frontier will be strictly convex from above. At any point on a *w(i)* frontier, the (absolute) slope of the tangent will show the capital–labour ratio, *k*, and the tangent will intersect the *w* axis at a point showing the net output per unit of labour, *y*. As *i* rises (and *w* falls) *k* and *y* will both fall and d*y*/d*k* = *i* (because *k* = -(d*w*/d*i*)). Naturally, $(d^2y/dk^2) < 0$. For each type of machine, then, there is a conventional *y*(*k*) production function relationship, with (d*y*/d*k*) = *i* everywhere.

With two kinds of machine, let their *w*(*i*) frontiers intersect for just one positive (*i*, *w*). The economy's production function will now consist of three sections. The first part, starting from *k* = 0, will coincide with that of the machine used at very high rates of interest, while the third part will coincide with that of the machine used at very low rates of interest. The second, intermediate, section will be a straight line, which is tangential to both the first and the third section and has a slope *equal* to the rate of interest at which there is a switch from one type of machine to the other. On this section, too, (Δ*y*/Δ*k*) = *i* but here this is simply an accounting relationship. Nevertheless, we have a Surrogate Production Function.

Now consider the possibility—which could never arise in Samuelson's case—that our two *w*(*i*) frontiers intersect for *two* positive (*i*, *w*) combinations. The production function now has five sections, as in figure 1: in order, a curved section; a straight line section corresponding to the switch point with the higher *r*; a second curved section relating to the 'other' type of machine; a second straight line section corresponding to the lower switching rate of interest; and, finally, a third curved section corresponding to *the same type of machine* as the first section. We have a Surrogate Production Function for which a given *type* of machine is used at both high and low rates of interest but *not* at intermediate levels of the interest rate. There is a 'return of the machine type'. It is striking that this kind of behaviour in a Surrogate Production Function results from *introducing* conventional substitution possibilities. (For further discussion of this kind of on/off use of inputs, see Opocher and Steedman, 2013.)

Figure 1. A Surrogate production function with return of machine type.



Source: our elaboration.

It should perhaps be said explicitly that *no* 'reswitching' is involved here. For the type of machine that returns, the machine–labour ratio will be lower for all the high interest rates at which it is used than it will be for any of the low rates at which it is in use. No given machine–labour ratio appears at more than one interest rate and thus, by definition, there is no reswitching. No reswitching, i.e. in the standard sense. But of course there is a family resemblance between our recurrence of on/off machine types and what Burmeister and Dobell once referred to as reswitching in a 'different sense' (Burmeister and Dobell, 1970, p. 279).

3. An example with infinitely many machine types

It is straightforward to extend the above argument to the case of infinitely many types of machine. For example, suppose that for every type of machine the $w(i)$ frontier is of the (restricted) 'Diewert' Quadratic Square Root form

$$a(i+w)+2b\sqrt{iw}=1 \tag{2}$$

machines now being everlasting for simplicity.
Let 'a' vary continuously in 0 ::: a ::: a^* and let b be a function of 'a' such that

(i) $b'(a) < 0 < b''(a)$
(ii) $|b'(a)|$ increases without limit as 'a' tends to zero
(iii) $b'(a^*) = -1$ and $b(a^*) > 0$.

To maximize w for given i (or vice versa), in (2), it will be necessary and sufficient to set

$$(i+w)=2\sqrt{iw}\,|b'(a)|$$

or

$$|b'(a)|=\frac{1}{2}\left[\sqrt{\frac{w}{i}}+\sqrt{\frac{i}{w}}\right] \tag{3}$$

When $i = w$ the right-hand side of (3) is unity and thus $a = a^*$.

But as (i/w) *either* falls *or* rises from unity, the right-hand side of (3) increases without limit. Since the left-hand side of (3) is monotonically decreasing in 'a', then, 'a' must fall (whether i/w has fallen or risen). Consequently, *every* value of 'a' in $0 \leq a \leq a^*$ will correspond to *two* distinct values of (i/w)—and hence of i. With the sole exception of the machine type represented by a^*, *every one* of the infinitely many machine types *returns* as i rises from zero around our Surrogate Production Function. Once again, of course, no reswitching of techniques is involved here.

4. Land, labour and different machine types

We now extend the surrogate model to include land, while returning to the assumption of circulating capital. We retain the assumption of 'equal proportions' in machine production and in the production of the consumer good. Indeed, we *strengthen* that assumption by supposing equal 'machineland' ratios as well as equal 'machine–labour' ratios.

Rather than again considering the possibility of a return of the machine type at different rates of interest, we now examine the reaction of *land and labour* use in the consumer good industry to a change in the wage/rent ratio.

For simplicity we shall suppose, unlike Samuelson, that the rate of interest is always zero. (A constant positive rate of interest would make no difference.)

Since the type of machine is qualitatively different for each technique, there is no loss of generality in stipulating that, for every technique, one machine is used in the production of one unit of the consumer good. If the latter process also uses l units of labour and t units of land then the input–output matrix can be written as

Table 1. A corn-machine model with land and labour.

	Machine	*Corn*
Machine	a	1
Corn	0	0
Labour	al	l
Land	at	t

Source: our elaboration.

It is clear that the price of a machine in terms of corn is simply '*a*', so that we have

$$lw + tr + a = 1$$

as the wage-rent frontier.

With infinitely many alternative techniques, suppose that

$$a = 2 - 6l + 6l^2 \tag{4}$$

and

$$t = l^2 \tag{5}$$

were $\sqrt{6} \le l \le 2$. At a given wage rate, w, the rent is

$$r = \left[\frac{-1 + (6 - w)l - 6l^2}{l^2} \right] \tag{6}$$

and this is maximized when

$$l = \left(\frac{2}{6 - w} \right) \tag{7}$$

Thus as w *rises* from zero to its maximum value of $\left(6 - 2\sqrt{6}\right)$, at which $r = 0$, l *rises* monotonically from (1/3) to (1 6). The greater is the real wage rate, the greater is the direct use of labour per unit of output in the consumer goodindustry, even with strong 'equal proportions', no reswitching and $i = 0$.

It can also be shown from (4) to (7) that the wage-rent frontier is $w = 6 - 2\sqrt{6 + r}$ and that, as w rises, t increases, t/l increases and a falls. We conclude our examination of this example by asking whether the possibility of unconventional outcomes at the level of the consumer good industry turns on the possibility of such outcomes at the level of the whole economy. It does not. Let C be the total output of the consumer industry and (L, T) be the total amounts of labour and land used in the economy as a whole (not just in the consumer good industry). It is readily shown that

$$\left(\frac{C}{L} \right) = 6 - 6\left(\frac{T}{L} \right) - \left(\frac{L}{T} \right) \tag{8}$$

While (8) is not, perhaps, a familiar form of production function, it does have the required properties. As (T/L) rises, output per worker, (C/L), rises. (C/L) is always increasing, but at a diminishing rate, and the derivative of (C/L) with respect to (T/L), in (8), is always equal to r—moreover, the tangent to the curve cuts the (C/L) axis at w. Hence (8), defined over the relevant range for (T/L), is a 'well-behaved' aggregate production function.

Thus an economic system that is conventional at its aggregate level can display quite unconventional relationships at the level of a particular industry. The reader may find some similarities between this broad result and that which Levhari (1965, p. 103) aimed to find, in a different model without on/off use of inputs, when he argued that 'Ruth Cohen's curiosum' may occur in an individual industry and *not* in the economy as a whole.

(Note that everything said in this Section could be interpreted as referring to a literally one-commodity economy, in which a is the own-use capital– gross output ratio. Under this interpretation, indeed, it is easier to see how a transition could be made from one technique to another.)

5. Concluding remarks

Starting from Samuelson's 1962 Surrogate Production Function, we have found here that some such functions are perfectly consistent with a 'return of the machine type'. And it is interesting that it is the *introduction* of a conventional possibility—that of machine–labour substitution—that makes possible this behaviour. We have also shown that, in the presence of *two* primary inputs, the surrogate model can give distinctly *unconventional* primary input use/input price relations at the level of the consumer good industry, even if the rate of interest is identically null.

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References

Burmeister E., Dobell A. R. (1970), *Mathematical theories of economic growth*, MacMillan, London.

Levhari D. (1965), *A nonsubstitution theorem and switching of techniques*, The Quarterly Journal of Economics, 79(1), 98-105.

Opocher A., Steedman I. (2013), *Full industry equilibrium. A theory of the industrial long run*, mimeo.

Samuelson P. A. (1962), *Parable and realism in capital theory: the surrogate production function*, Review of Economic Studies, 29(3), 193-206.

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Edgardo Bucciarelli[*] - Nicola Mattoscio[**] - Michele Alessi[***]

WHAT IS ESSENTIAL IS INVISIBLE TO THE EYE. FROM NEOCLASSICAL TO COGNITIVE PERSPECTIVE ON PRODUCTION THEORY

Abstract

This article is to be considered within the extensive contemporary debate between the neoclassical economic approach and the current heterodox economic approaches on the theory of production. Historically, this debate has been represented by the Cambridge-Cambridge controversy beginning with the problems of the heterogeneous nature of the capital factor and leading one to question a dogma that characterizes the neoclassical economic theory: the Cobb-Douglas production function. In 1974 and in 1980 Anwar Shaikh completely debunked this dogma by rendering visible to the scientific world the income accounting identity underlying Cobb-Douglas production function and thereby leading him, in 2012, to urge to rethink microeconomics. One of the current heterodox economic approaches which has addressed this urge, is the strand of studies that heads cognitive economics. According to cognitive economics, what is essential is the cognitive process since it is the origin and cause that determines those phenomena that economic science intends to analyse. In this article, we adopt the cognitive perspective in order to make analytically explicit the cognitive production function which describes the cognitive process, invisible to the eye, but that lies at the root of the production of knowledge required for developing and optimizing the production process itself.

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To achieve this aim, we rely on the joint anlysis of John Foster's economic thought, which is related to evolutionary economics, and on Max Boisot's economic studies on information economics, which should be seen as connected with the recent strand of studies of cognitive economics.

JEL Classification: A12, B21, B31, B52, D01, D21, D83.

Keywords: Anwar Shaikh's Identity, Cobb-Douglas's Dogma, John Foster, Evolutionary Economics, Max Boisot, Cognitive Economics, Cognitive Production Function.

1. Introduction

For at least a half a century we have been witnessing a well and proper fragmentation of economic theory that has led economics studies to diversify into various directions: be it methods and analytical techniques, be it the notions and assumptions as well as the nature of the economic phenomena referred to. Due to the coexistence of several lines of research in contemporary economic debate a clear contrast emerges between the perspective of the neoclassical theory (so called *mainstream*) and that of evolutionary economics and of the current heterodox economic approaches, such as the cognitive economics.

Focusing on the methodological approach adopted, we can observe that, on the one hand, the neoclassical economic theory is based on deductive reasoning while on the other hand, the current heterodox economic approaches are distinguished on account of their prevalence of inductive reasoning[1]. This means that neoclassical theory, which relying on assumptions and models that systematically remove the

[1] Among the current heterodox economic approaches, behavioral economics and cognitive economics are the most promising ones. Nevertheless it is to be underlined that, compared with the neoclassical economics, the behavioural economics distinguishes itself in drawing on empirical evidence as well as inductive reasoning to define the assumptions from which its models are deductively derived; however behavioural economics is similar to the neoclassical economics in relating the efficacy of its models to how such models are deductively derived from the assumptions themselves. In respect to the behavioural economics, not only does the cognitive economics try to innovate the neoclassical approach on how the assumptions should be formulated, but also it represents an attempt to innovate the neoclassical methodological approach by stressing the efficacy of adopting an interdisciplinary approach to investigating into economic phenomena (see, among others, Rumiati 1990, Rabin 1998, Camerer 1999, Innocenti 2010).

complexity of reality, comes to general and abstract conclusions, while the more significant current heterodox economic approaches, by using the inductive method, try to recover the complexity of reality ignored or removed by the mainstream, using, for example, the experimental method. Further to this methodological difference, there is also a difference in the perspective from which one observes, describes, and explains economic phenomena.

To observe economic reality[2] from a neoclassical perspective entails to oversimplify all those phenomena that are, instead, by their very nature complex and in continual evolution. The prevalent heterodox perspective, contrary to the neoclassical one, attempts to examine economic phenomena taking into consideration their complexity and in particular tries to recover the possibility to express the individual characteristics of human beings in theoretical terms. The neoclassical economics does not offer adequate tools for this purpose in that it has developed for more than a century on the need for simplification. As a consequence, the neoclassical perspective has not taken into consideration the plurality of individuals, but it has adopted one single impersonal anonymous individual as the representative economic agent[3] (Colander *et al*. 2004;

[2] Concerning the concept of "*economic reality*" see Mario Alberti (1938) and Harald Uhlig (2012).

[3] The neoclassical approach is based on the belief that the principle of generalization resulting in a simplification of the complexity makes it possible that the representative agent, *i.e.* homo œconomicus, sums up in herself all the relevant characteristics of individuals. More precisely, the representative agent follows behavioural rules mathematically and a priori established and adopts criteria of choices, called axioms, which are perfectly coherent (Colander 2000a, 2000b; Novarese and Rizzello 2004). As a consequence, the simplification of the reality on account of the adoption of the anonymous representative agent makes the neoclassical models blind to some of the most crucial aspects of economic science, *e.g.* information asymmetries, uncertainty, credit rationing, risk sharing, and the effects of variations in income distribution. Such neoclassical models are, moreover, unable to take account of the impact of the political economy on inequality as well as the effect of the inequality on the economic performance, as a consequence of underestimating how negatively economic instability affects human well-being. Furthermore, models adopting the representative agent provide descriptions which are incongruous in respect to the advanced industrial countries and insignificant in regard to the developing ones (Colander 2000b; Stiglitz *et al*. 2006). In conclusion, it is possible to sum up the core of the critiques concerning the neoclassical approach by making reference to the following concepts: bounded rationality (Simon 1959, 1978), systematic bias (Allais 1953), and the relevance of perceptions, expectations and motivations (Katona 1980). It is now clear that the current heterodox approaches, originated by the virtuous interaction between economics and psychology, stem especially from the critique pointing out the restraints and inadequacy of adopting the perspective of the representative agent to investigate into the complexity of economic phenomena. As a consequence, there is the increasing tendency among the contemporary economists towards asking themselves not how an anonymous ideal type representative agent should choose or act, but why individuals act according to

Camerer, Loewenstein and Prelec 2005). The tendency to oversimplify, which distinguishes neoclassical economic thought throughout its history, has not met with unanimous consent in the world of science, it has in fact been the cause of heated debates on the methodology to be adopted as well as on the nature of the object to be studied. If we take as a point of reference a milestone of modern economic thought, which is the capital factor crucial in the theory of production, what is said above is exemplified in the famous Cambridge-Cambridge controversy which caused a dialectical rift between Cambridge UK and Cambridge US in the Fifties and Sixties of the last century, regarding the problems of the heterogeneous nature of the capital factor, its meaning, its measurability, and consequently also regarding the production function. Historically this dialectical rift reflects the present existing debate on economic theory between the neoclassical mainstream and the current heterodox approaches. Now, as in those times, the economists are debating not on the economic phenomena to be observed but on their nature and on how to examine them. In this debate, the economic phenomena observed by us is part of the theory of production, and so our article aims to contribute to highlight the urge to rethink the economic theory of production using as a basis the central roles played in this theory by the cognitive process and cognitive assets.

Actually, according to the theory of production, information and knowledge play a central role in the complexity of the current productive processes (see also Nonaka 1991; Arrow 1996; Stiglitz 2000, 2002; Foray 2000; Nonaka and Takeuchi 2001; Rullani 2004; Nonaka and Toyama 2007) because they affect production decisions and purposes of innovation. This is on account of, they permit to make use of productivity and quality sources that were not visible previous to the adoption of the cognitive perspective. In light of the considerations on the role knowledge plays in the production process, it becomes utterly necessary to examine the cognitive process producing the knowledge required for developing and optimizing the production process itself. In

subjective, various and observable modalities in dealing with the complex economic reality.

this work, we focus in particular on the capacity of the Schumpeterian entrepreneur to produce the knowledge required to achieve the desired level of production with the highest degree of efficacy. Substantially, we define the analytical expression of the cognitive process, invisible to the eye, which produces the knowledge required for developing and optimizing the production process itself. To achieve this aim, we rely on the joint analysis of John Foster's and Max Boisot's economic thought. Our work examines, on one hand, John Foster's vision of the entrepreneur in Schumpeterian evolutionary terms[4] (2000, 2005), related to the current heterodox economic approach bof evolutionary economics; and, on the other hand, Max Boisot's economic studies on *information economics* (Boisot 1995, 1998, and Boisot and Li 2006), which should be seen as connected with the recent strand of studies of cognitive economics.

The article is structured in the following manner: in Section 2, thanks to Anwar Shaikh's identity, we show an impressive example of how a fundamental dogma of the neoclassical *mainstream* can be debunked: the *Cobb-Douglas production function*. In Section 3, bearing in mind Anwar Shaikh and relying on the joint analysis of John Foster's and Max Boisot's economic thought, we define the epistemological foundation, which is applied in Section 4, rendering it possible to develop and make analytically explicit Max Boisot's *cognitive production function*. Finally, in Section 5, in light of what we have discussed in the article and in view of future empirical applications, we offer final considerations.

2. Anwar Shaikh's identity and the Cobb-Douglas's dogma

"*It has been related further that on the same day the madman forced his way into several churches and there struck up his requiem aeternam deo.*"
(Nietzsche F. W., 1883, Aphorism 125, The Gay Science)

The contribution of Charles W. Cobb, Paul H. Douglas and Anwar Shaikh to the study of economics are significantly different: the first two

[4] On reconsideration of the Schumpeterian evolutionary terms, see Kirzner (1999), Reinert (2002) and Witt (2002).

founded a dogma of the *neoclassical faithful*[5], while the latter contributed to decisively debunking that dogma, and so to rethinking economics starting from a perspective considering the complexity of reality.

In their 1928 article "*A theory of production*", Cobb and Douglas, starting from a short time series of data (1899-1922) on the American manufacturing industry, introduced that which became a dogma of neoclassical economics: the Cobb-Douglas production function. In 1956, twenty-eight years later, the Cobb-Douglas[6] production function was consecrated by Robert Solow in his work "*A contribution to the theory of economic growth*", in which it was one of the foundamental assumptions of that which became the model[7] of reference for the theory of economic growth[8] and the starting point for those studies that would see him awarded the Nobel Prize for economics in 1987. In 1976, Paul H. Douglas, in his article "*The Cobb-Douglas production function once again: its history, its testing, and some new empirical values*", returned on the subject with strong conviction to restate the validity and the scientific significance of that which had been stated almost a half a century earlier by him with Cobb[9], ignoring all those theoretical and empirical scientific contributions, which beginning with Joan V. Maurice Robinson[10] (1953), demonstrated the scientific limitations of

[5] The expression "*neoclassical faithful*" is used here as adopted by Anwar Shaikh (1980).

[6] More specifically, it is there considered the aggregate Cobb-Douglas production function.

[7] It is to be underlined that also other models, which are historically representative of the theory of economic growth, included the Cobb-Douglas production function within their assumptions; among others Kaldor (1956) and Swan (1956).

[8] For a survey on the various strands of research stemming from Solow's work dated 1956 the reader is referred to Solow (1974, 2000) and for a meaningful critique on them to Pasinetti (2000).

[9] Cfr. Douglas (1976): "*This paper is an effort to continue and extend earlier studies of the production function that were first begun nearly a half century ago*." (p. 903); "*I felt, therefore, that the marginal productivity theory of wages had received a substantial degree of confirmation.*" (p. 904). "*Many of the original objections have been answered. Some remain.*" (p. 914).

[10] "*The production function has been a powerful instrument of miseducation. The student of economic theory is taught to write Q = f (L, K) were L is a quantity of labor, K a quantity of capital and Q a rate of output of commodities. He is instructed to assume all workers alike, and to measure L in man-hours of labor; he is told something about the index-number problem in choosing a unit of output; and then he is hurried on to the next question, in the hope that he will forget to ask in what units K is measured. Before he ever does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next*." (Robinson 1953, p. 81).

his original statement. That which will become the *Cambridge capital theory controversy* starts with Joan Robinson's critique of the meaning and measurability of capital factor and becomes the wider critique of the aggregate production function. Among the most significant critics of the famous controversy we note along with Joan Robinson, Pasinetti (1959), Piero Sraffa (1960), Pierangelo Garegnani (1960, 1970) and, for a comprehensive critical review refer to the eminent economist Geoffrey Harcourt (1972). Notwithstanding the criticisms levelled at the Cobb-Douglas function as well as the more recent and radical scientific examination of it (see, among others, Sylos Labini 1995, 2001) "*the Cobb-Douglas function also continues to be accepted as a sort of dogma in recent works concerning both advanced and underdeveloped countries, often attributing to the exponents more meaning than the original ones* (Sylos Labini 2001, p. 84).

In 1974, Cobb-Douglas's dogma, founded in 1928 and consecrated in 1956 in the Solow's model, was debunked in its foundations by Anwar Shaikh throughout its seminal work "*Laws of production and laws of algebra: the humbug production function*" in which he introduced his fundamental identity:

$$q(t) \equiv w(t) + r(t)\, k(t)$$

"*where q(t) and k(t) are the output-labor and capital-labor ratios, respectively, and w(t) ≡ W(t)/L(t), r(t) ≡ π (t)/K(t) are the wage and profit rates, respectively.*" (Shaikh 1974, p. 116). Actually, Shaikh, starting from this identity and through a series of simple algebraic transformations which yields to the following algebraic relation

$$Q = B(t)[c_0 K^{\beta} L^{1-\beta}]$$

demonstrates that the Cobb-Douglas form "*is a mathematical relationship, holding true for large classes of data associated with constant shares, it cannot be interpreted as a production function, or any production relation at all.*" (Shaikh 1974, p. 117), conclusion which is reiterated in his work of 1980: "*Once again it would seem that the apparent empirical* *success of the Cobb-Douglas function having 'correct' coefficients is perfectly consistent with wide variaties of data, and cannot be interpreted as supporting aggregate neoclassical production and distribution theory.*" (Shaikh 1974, p. 119; 1980, p. 92). Hence, for the first time in history, Anwar Shaikh renders visible what was invisible to the international scientific world up until that time: there is an income accounting identity which underlies the Cobb-Douglas production function and therefore the Cobb-Douglas production function is no more than a mathematical relationship which merely resembles a production function. In fact, thanks to his seminal contributions in 1974 and 1980, Shaikh succeeded in demonstrating how one instrument of the neoclassical theory of production does not adhere to reality: "*The analysis of the laws of algebra led to the conclusion that any production data series q, k whatsoever, can be represented as being generated by a Cobb-Douglas production function having neutral technical change and satisfying marginal productivity 'rules', so long as shares are constant and the measures of capital and labor such that k is uncorrelated with* $\dot{B}/B$." (Shaikh 1980, p. 85).

Moving our reflection on Cobb-Douglas's dogma from an algebraic level to an epistemological level of analysis, and from the laws of algebra to the laws of thermodynamics, we can surmise that if an income accounting identity underlies the Cobb-Douglas form, then, this form can be written as a balance equation, a typical example of which is given by the analytical expression of the first law of thermodynamics. A balance equation allows to effectively describe biophysical phenomena characterized by the conservative nature of the energy within their systems. But social systems, and so too the economic systems, are not conservative in nature. As a matter of fact, if we take as an example the production process in itself, it becomes evident that not all inputs can be transformed into outputs, so much so that not only are production techniques and organizational structures sought to reduce the loss of energy in production cycles but also to reduce waste materials (*e.g. statisticial process control techniques*), as described by Walter A. Shewhart's work dated 1931. It is noteworthy that John M. Keynes in his "*General*

Theory" (1936) does not make use of the production function as a fundamental analytical tool (Foster 2006), and Williamson too states that firms are not to be conceived as production functions, but as *alternative governance structures* (Williamson 1985, 2002).

With Anwar Shaikh's theoretical and empirical demonstration that Cobb-Douglas is a *humbug function* (1974, 1980)[11] numerous other scholars have fed the debate on the critique of the production function (among these, see McCombie and Dixon 1991; Felipe 2001; McCombie 2001; Felipe and Holz 2001; Sylos Labini 2001; Felipe and Fisher 2003; Fisher 2005; Felipe and Adams 2005; Felipe and McCombie 2005, 2009; Pressman 2005, Carter 2011). One such example is Felipe and Holz (2001) who show that "*the Cobb-Douglas form is robust to relatively large variations in factor shares. However, what makes this form quite often fail are the variations in the growth rates of the wage and profit rates. The weighted average of these two growth rates has been shown to be the coefficient of the time trend. This implies that, in most applied work, a Cobb-Douglas form (i.e. approximation to the income accounting identity) should work. We just have to find which Cobb-Douglas form with a dose of patience in front of the computer.*" (Felipe and Holz 2001, p. 281).

The debunking of one of the main dogmas of neoclassical economics, the humbug Cobb-Douglas, has allowed to highlight new possible paths that lead to recover the complexity of reality for contemporary economic thought. In fact, if it is true that the Cobb-Douglas production function was conceptualized from a specific time series of data collected from reality, it is, then equally true that reality changes according to the analytical tool which is used to observe it, and, therefore, the criterion for choosing the instrument with which to observe reality cannot be exclusively one of readiness, but must be first and foremost that of efficacy. And, as Sylos Labini recalls, "*then its [Cobb-Douglas production function] adoption has negative effects in the interpretation of the growth process, since it can divert the analytical efforts of the economists into false directions.*" (Sylos Labini 2001, p. 84).

[11] Phelps Brown (1957) and then Simon and Levy (1963), Fisher (1971), Samuelson (1979), and Simon (1979) are also to be considered.

Anwar Shaikh himself, about forty years after the publication of his seminal work dated 1974 "*Rethinking microeconomics*" (2012), shows that he has not fallen into the trap of nihilism, but has come to shed the light of the lantern on new paths which lead to rethink economics and recover the inner person: "*There is a great difference between studying how people actually behave and positing how they should behave. When we wish to know how and why people behave as they do, we turn to behavioral economics, anthropology, psychology, sociology, political science, neurobiology, business studies and evolutionary theory. We discover that evolutionary roots, cultural heritages, hierarchical structures and personal histories all influence our behavior: we are socially constructed beings, within the limits of our evolutionary heritage (Angier 2002; Ariely 2008, Ch. 4-5, 9; Zafirovski 2003). There is a large body of evidence which shows that we do not consistently order preferences, we are poor judges of probabilities, we do not address risk in a "rational" manner, we regularly commit a wide variety of reasoning errors, and we generally base our behavior on habits and rules of thumb (Agarwal and Vercellli 2005, p. 2; Anderson 2000, p. 173; Conlisk 1996, pp. 670-672; Simon 1956, p. 129). […] Despite all of this evidence, neoclassical economics stubbornly insists on portraying individuals as egoistic calculating machines, noble in reason, infinite in faculty and largely immune to outside influences. The introduction of risk, uncertainty and information costs change the constraints faced, but not the basic model of behavior (Furnam and Lewis 1986, p. 10). I will call this the doctrine of "hyperrationality" so as to distinguish it from a more than the general notion of "rationality" which refers to the belief or principle that actions and opinions should be based on reason. The point here is to avoid the neoclassical habit of portraying hyperrationality as perfect and actual behavior as imperfect. It is a topsy-turvy world indeed when all that is real is deemed irrational*." (Shaikh 2012, pp. 3-4). And as remarked by Hugo Reinert and Erik S. Reinert: "*We are living at a time where standard neoclassical economics is entering a period of decline. In order to achieve any degree of relevance, whatever theory replaces this mechanical and barter-based view of economic will have to incorporate Nietzschean traits: without Man's wit and will, his incessant creative process, and the role of the human beings who push this forward, economics will – as neo-classical economics – always be like playing Hamlet without the Prince of Denmark.*" (Reinert and Reinert 2006, p. 79).

From this comes the urge to recover the real human being for economic studies. This forces one to take into consideration, among other things, human cognitive activity which lies at the root of the "*incessant creative process*" from which follows the evolution of economic systems (Foster and Metcalfe 2001). With reference to the theory of production, this entails taking into account and analyzing the cognitive activity which allows to optimize the production process. In other words: "*this value does not come just from the elements contained in the firm – the individuals, the machines, etc. – but from the connections that are forged between them (see Shapiro and Varian 1999).*" (Foster 2005, p. 885). As a consequence, it is important to focus on the cognitive process which lies at the root of the production of new knowledge required for finding those *combinations* permitting to optimize the organization of the production process, among other things. More specifically, in the following two sections, relying on the joint analysis of John Foster's and Max Boisot's economic thought, we render it possible to describe analytically the cognitive process that allows human beings to produce the knowledge required for achieving their aspirations while adapting to the continual evolution of the environment where they interact.

3. What is essential is invisible to the eye

"*What is essential is invisible to the eye*"
(de Saint-Exupéry A. [1943] (1995) p. 82)

In this section, we jointly analyze John Foster's and Max Boisot's economic thought so as to define the epistemological foundation which renders it possible to develop and make analytically explicit Max Boisot's *cognitive production function* describing the human cognitive process. Referring to the title of this Section, what is essential are the cognitive assets[12] and, more precisely, the cognitive process which is at the root of the production of the knowledge required for achieving one's aspiration. This cognitive process is invisible to the eye in that is takes place at a mental level. More specifically, we focus on the cognitive process that produces the knowledge for planning,

[12] On the concept of *cognitive assets* see Eden and Spender (1998), Fiol (2002), Lant (2002), Cataldo and Prochno (2005), and Aoki (2010).

developing and optimizing the strategy useful to gain the desired goal considering, at the same time, the complexity of the biophysical and socio-economic environments where one interracts. We deal with the cognitive process which distinguishes the *economic agent* who we call *cognitive agent*[13]. In fact, the term cognitive refers to the wider cognitive paradigm, and more precisely, to a way of analyzing psychic phenomena which consists of representeing and interpreting them as mental representations caused by the elaboration of information extracted from data (Dosi *et al.* 1999; Castelfranchi 2003). Nevertheless the cognitive paradigm does not exclusively include those scientific activities that study cognitive processes, but it also refers to the studies of all those phenomena that involve the cognitive activities of the human mind to process data, keeping well in mind that human mind is not necesserly rational, and therefore cognitive does not stand only for rational[14].

The *cognitive agent* is forced to face a complex reality to which she has to produce an adaptive response. In dealing with the existing complexity, her response has more or less efficacy as to her aspiration. Since her complexity of reality is in continual evolution, the adaptive response of the *cognitive agent* has to vary with time. In fact, the *cognitive agent* is also defined as such for the very reason that she is able to adapt to the complexity of the biophysical and socio-economic environments where she interacts in her context producing adaptive responses with varying degrees of efficacy. Applying such a line of thinking to the

[13] In scientific literature, the term *cognitive agent* refers mainly to an individual endowed with mental representations which guide her. In particular, she uses tools and formats, which include images, propositions and schemes, in order to make mental representations both of her aspirations as well as intentions, and of the biophysical and socio-economic environments where she interacts; moreover, the cognitive agent is invested with beliefs and opinions and she is placed in a complex context structurally pervaded by uncertainty (Patalano 2005). In our work, we make use of the expression cognitive agent so as to shift the focus from the hyperrationality distinguishing the representative agent of the economic mainstream to the cognitive activity underlying the actions of the cognitive agent of the Cognitive Economics. In fact, this latter approach highlights how economic behaviour differs from agent to agent in that it is related to the specific byophisical characteristics of each agent interacting in an evolving complex environment. Thus, the focus is on the cognitive capacity of the individual to interconnect elements of the complex reality in which she interacts in a such a way that she can achieve her aspiration with the highest degree of efficacy.

[14] Indeed, rationality is a particular way of operating of the cognitive apparatus that occurs whenever beliefs are rooted on sufficient and convincing evidences, when inferences are not tricked by distorsions or illusions, and decisions are based on a correct evaluation of expected risks and advantages.

theory of production[15], what is invisible to the eye is the cognitive process that produces the knowledge required for developing and optimizing the production process, that is the optimization of the combination of the available factors (human beings and physical object) to achieve the desired level of production. And so, the focus is shifted from production factors themselves to how they are combined (Loasby 1999).

In line with Foster, such an optimization can be defined subjective in that it depends on the specific cognitive activity of the individual cognitive agent (Foster 2005, pp. 882-883), and consequently, the production process can be defined in terms of *self-organization* on account of the fact it can be also interpreted as seeking new combinations of the available production factors with an higher degree of efficacy in terms of the production of the desired level of output (Foster 2000). In seeking new more effective combinations, the cognitive agents forced to face the complexity of the biophysical and socio-economic environments[16]. Faced with such a complexity, the *cognitive agent* has to respond to it by producing the best adaptive response which, in the economic context dealt with here, means optimising the production process while considering not only the initial endowment of production factors but also the complexity of the biophysical and socio-economic environments. The biophysical and socio-economic environments are not stationary but in continual evolution. Therefore, the *cognitive agent* has to adapt in time the combination of her production factors taking into account the evolution in both her production factors and in the biophysical and socio-economic environments[17].

[15] Following neoclassical theory of production, which analyzes the production or the economic process of converting input into output, we here refer to the cognitive process underlying the combination of production factors from the evolutionary approach (Nelson and Winter 1978, 1982).

[16] The environmental complexity is expressed in its biophysical, social, cultural, institutional and economic aspects.

[17] This suggests the reason why "*standard economics would develop a simple analytically-solvable function -say the CES production function with 'nice' analytic properties - and then use that to study a variety of cases. In the complexity approach, one would try hundreds of variations of non-linear models, many with no deterministic solution, and rely on the computer to show which model best fits the data. One would,*

In this context, the most fitting example of the cognitive agent is that of Schumpeterian entrepreneur (see, among others, Foster 2000). In truth, Foster analyzing Schumpeter's economic thought emphasizes that "*it is clear that the development processes that he [Schumpeter] discusses are concerned with organisational change at several levels and these are orchestrated by entrepreneurship, which is a behavioural attribute that only has meaning in organizational contexts: everyone is an entrepreneur only when he actually 'carries out new combinations' and loses that character as soon as he has built up his business, when he settles down to running it as other people run their business (Schumpeter 1934, p. 78). The entrepreneurial desire to discover new and profitable organsational combinations provide what we can now label as a self-organisational impetus within the economic system, creating organized complexity.*" (Foster 2000, p. 319). In fact, the Schumpeterian entrepreneur so as to achieve the levels of production aspired to, not only is she asked to have a certain initial endowment of production factors, but she is also required to find the best possible combination of those production factors at the same time considering the continual evolution of the biophysical and socio-economic environments (Foster 2005, p. 882), the combination she finds is defined as the adaptive response of the Schumpeterian entrepreneur. Thus, such levels of production also depend on the entrepreneur's subjective capacity to produce an adaptive response with varying degree of efficacy, as revealed by the following:

1. how she has combined her own initial endowment of production factors in order to achieve her aspiration;

2. how she has adopted her production process to the complexity and continual evolution of her own initial endowment of production factors and of the biophysical and socio-economic environments[18].

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of course, study the general properties of these non-linear models, but whether the models have analytic solutions would not be a relevant choice criterion as it is now; the choice criterion would be 'fit with the data'. Elegance and solvability of models are de-emphasized. [...] As computing costs continue to fall, analytic solutions in mathematics will be less and less important. Similarly in econometrics; Monte Carlo and bootstrap methods will replace analytic methods of testing in many cases." (Colander 2000b, pp.5-6)

[18] With reference to Section 2, and *i.e.* to the algebric identity underlying the Cobb-Douglas production

Thanks to Max Boisot's economic studies on *information economics* (Boisot 1995, 1998; Boisot and Li 2006), we can, at this stage, define the cognitive process of the *cognitive agent*, that process which lies at the root of the production of the knowledge required for planning, developing and optimizing her strategy to achieve her aspiration, and in that same process considering, at the same time, the complexity of the biophysical and socio-economic environments where she interracts. The strategy implemented by the *cognitive agent* is her adaptive response to the complexity of the reality with which she is faced in order to achieve her goal. To conceive such an adaptive response. For this to happen, the *cognitive agent* has to produce new knowledge. Indeed, "*knowledge is a property of agents predisposing them to act in particular circumstances.*" (Boisot 1998, p. 12). To produce the knowledge required for conceiving her own adaptive response, the *cognitive agent* has to process the data available to her. As a matter of fact, the complexity of the reality that the *cognitive agent* is forced to face can be seen as an amount of data since "*[…] data can be treated as originating in discernibile differences in physical states of the world - that is, states describable in terms of space, time, and energy […]. Agents are bombarded by stimuli from the physical world not all of which are discernable by them and hence not all of which register as data for them. Much neural processing has to take place between the reception of a stimulus and its sensing as data by an agent (Kuhn 1974)*" (Boisot and Canals 2004, p. 46-47). Thus, among all the amount of data that the cognitive agent can process, not all of them are relevant as among the data not all data is information. In fact, information is only that data "*that modifies the expectations or the conditional readiness of an observer. The more those expectations are modified, the more informative the data is said to be*" (Boisot 1998, p.20); therefore, " *[...] information, to be sure, is something that is extracted from data in order to modify knowledge structures taken as dispositions*

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function, we can deduct that all the potential levels of output do not exclusively depend on the amount of input factors but also on the entrepreneurial capacity, as defined above, of adopting in an optimal way the production process to the extent to which the biophysical and socio-economic environments change. As noted by Foster (2005, p. 885) bearing in mind Williamson (1985): "*[…] firms are governance structures, not production functions.*".

to act in order to modify knowledge structures taken as disposition to act. [...] Information, to be sure, retains an essential role as the product of extractive operations designed to economize on the consumption and processing of data." (Boisot 1998, p. 26).

For this reason, the cognitive agent in selecting data from which to extract information for the production of knowledge required for developing and optimizing the production process, is forced to economize on their comsumption[19]. From thus it is deductible that the cognitive agent so as to economize on the consumption of the available data, not only does she have to distinguish those data relevant to her aim, but she also has to create categories to which assign the data, and then, she also has to reduce them to the most suitable categories for her goal. Such a cognitive activity of creating categories to which assign the data and of reducing those categories is the cognitive activity of articulation. By cognitive activity of articulation we mean the cognitive activity of coordinating the interrelated activities of codification with that of abstraction.

We now examine the details of these specific cognitive activities. The cognitive activity of codification "*creates categories in order to make clear and reliable distinctions between relevant states of the world that one can act upon—between black and white, between heavy and light, between right and wrong, between cases that will be reimbursed by one's medical insurance and those that will not, and so on. In creating and clarifying categories, codification aims to take the fuzziness out of phenomena; it is analogous to de-fuzzifying a fuzzy set (Pedrycz and Gomide 1998). Codification entails time-consuming data-processing efforts, and the larger and more complex the number of categories that one has to deal with, the greater the data-processing effort involved. Where*

[19] Boisot reconducts the action of economizing to the Principle of Least Action (PLA) defined for the first time by Pierre Louis Maupertuis in 1744: "*Maupertuis's principle of least action, namely, the idea that physical processes in general so organize themselves as to minimize their consumption of a quantity known as the action. If we replace the term action by energy, Maupertuis's still holds good – Knowledge allows us to minimize the use of energy (Energy economics); Knowledge allows us to minimize the action of processing data (through codification and abstraction) aimed at extracting information from data in order to gain new knowledge, that is evolution (Information economics)*." (Boisot 1998, p. 15).

codification is effective, it allows one to subsequently assign phenomena unproblematically to a given set of categories." (Boisot and Li 2006, pp. 226-227). Therefore, the degree of efficacy of the cognitive activity of codification depends on the cognitive agent's subjective cognitive potential for codification. Actually, each cognitive agent has her own cognitive potential for codification which she can use to a higher or lower degree of efficacy in the short run and which she can improve in the medium-long run by *learning*[20] and/or using specific *artefacts*.

The cognitive activity of abstraction " *[...] gives them* [phenomena] *structure. If codification allow us to save on data-processing resources by allowing us to group the data of experience into categories, abstraction allows us to realize further savings in data processing by minimizing the number of categories that we need to draw on for a given task. [...] When properly carried out, abstraction allows one to focus on the structures, causal or descriptive, that underlie the data."* (Boisot 1998, pp. 49-50). The degree of efficacy of the cognitive activity of abstraction depends on the cognitive agent's

[20] The concept of learning normally alludes to the peculiar change for a particular agent within a particular environment (Toates 2007). As many authors have pointed out, the continuing transformations of manufacturing processes and the related production cycles have raised discussions about the nature of learning in modern organizations, as well as about the role of knowledge and the methods of learning and development required to deal with new advanced forms of production (among others see Lundvall 1985, 1988; Adler and Cole 1993, 1995; Tapscott 1996; Kyrö and Enquist 1997; Aoki 2006; Lundvall and Vinding 2004; Pihlaja 2005; Vernengo 2010). As early as a quarter of a century ago, Shoshana Zuboff pointed out that learning had become a pillar of every day work. "*Learning is the new form of labor. It is no longer a separate activity that occurs either before one enters the workplace or in remote classroom settings [...] Learning is the heart of productive activity*." (Zuboff 1988, p. 395). Therefore, learning is not about how much the cognitive agent knows, but how effectively she processes or handles the information received. The acquisition and processing of information by mental or cognitive processes lead learning to become cognitive learning that is reflected in organized and new knowledge. When the concept of learning is linked to that of the Schumpeterian entrepreneur, it is possible to describe learning as a continuous process that facilitates the development of necessary knowledge for showing a high degree of efficacy in starting up and managing new ventures, and in how to make innovation and how to overcome traditional production obstacles (Ronstadt 1988; Shane and Venkataraman 2000; Sheperd *et al.* 2000; Corbet 2002). As a result, "*while traditional microeconomics tends to focus upon decisions, made on the basis of a given amount of information, we shall focus upon a process of learning, permanently changing the amount and kind of information at the disposal of the actors. While standard economics tends to regard optimality in the allocation of a given set of use values as the economic problem, par préférence, we shall focus on the capability of an economy to produce and diffuse use values with new characteristics. And while standard economics takes an atomistic view of the economy, we shall focus upon the systemic interdependence between formally independent economic subjects.*" (Lundvall 1988, p. 349).

cognitive potential for abstraction. The cognitive potential for abstraction allows the *cognitive agent* to economize on data processing:

"*Abstraction, in effect, is a form of reductionism; it works by letting the few stand for the many.*" (Boisot 1998, p. 50). Such cognitive potential for abstraction is by its very nature subjective in that the cognitive agent can use it to a higher or lower degree of efficacy in the short run, and which she can improve in the medium-long run by learning and/or using specific artefacts.

We can therefore conclude that the cognitive activity of articulation consists of coordinating the interrelated activities of codification with that of abstraction. As Boisot stresses referring to Ulrike Hahn and Nick Chater (1998a, 1998b) "*articulation will be efficient in its use of resources to the extent that the categories that it draws upon to capture some phenomenon are distinct from each other (i.e., well codified) and few in number (i.e., abstract).*" (Boisot and Li 2006, p. 229). Hence, the *cognitive agent* can minimize the amount of data distinguishing and capturing those to be processed for the production of the required new knowledge thanks to the coordination of the interrelated cognitive activities of codification and abstraction. In conclusion, the capacity for minimizing the amount of data depends on the *cognitive agent*'s cognitive potential for articulation which is given by the coordination of the interrelated activities of codification with that of abstraction. Consequently, the more the *cognitive agent* increases her cognitive potential for articulation, through the relative increase for her potential for abstraction and for her cognitive potential for codification, the more the amount of data to be processed decreases. The *cognitive agent* can use her own potential with varying degrees of efficacy in the short run, and improve it in the medium-long run thanks to learning and/or the use of specific *artefacts*. With Ghisellin and Landa (2005) in mind as quoted by Boisot and Li (2006), we can take as an example the hypothetical case in which the *cognitive agent* is a Schumpeterian entrepreneur who has to distinguish those who are trustworthy from those who are *free riders*. In our example, the Schumpeterian entrepreneur's potential trading partners stand for the amount of data to be processed by her. The potential trading partners can be assigned to specific categories. The

Schumpeterian entrepreneur uses her cognitive activity of codification both for creating categories and for assigning her potential trading partner to one of them. Indeed, our Schumpeterian entrepreneur could use different criteria to create categories to which she assigns her potential trading partners. Such criteria are determined by the efficacy they have in achieving the goal the entrepreneur sets herself. To determine which criteria of assignment has the most efficacy, the entrepreneur has to use her cognitive activity of abstraction. Not only does this cognitive activity of abstraction allow her to distinguish the criterion of classification with the most degree of efficacy for achieving her goal, but it also allows her to reduce the number of categories to which she assigns her potential trading partners to a number that can be easily processed at a cognitive level, *i.e.* according to George A. Miller (1956), the famous number seven plus or minus two. To each category distinguished on the basis of the criterion previously identified there is a definite degree of trustworthiness for trading partners. More precisely, the degree of trustworthiness for each categories depends on the underlying criterion used to create the categories. In conclusion, with the cognitive activity of articulation (coordination of the interrelated activities of codification and abstraction) the entrepreneur is able to assign her potential partners to categories on the basis of the criterion she has identified according to her goal: defining with the least effort possible both the degree of trustworthiness of each new trading partner and the optimal strategy to relate with them. The *cognitive agent*'s cognitive process so far described, is made analytically explicit by us in the following Section by developing Max Boisot's pre-analytical notion of the *cognitive production function* into an analytically explicit expression.

4. Cognitive production function

In Section 3, we introduced, thanks to the joint analysis of John Foster's and Max Boisot's economic thought, the concepts of cognitive agent and cognitive process which distinguishes her, and described in more detail the cognitive process that produces knowledge. As a matter of fact, the cognitive agent, in order to pursue her aspiration has to produce her adaptive response for the most efficacy in facing the continual evolution of the complexity of the biophysical and socio-economic environments where she interacts (Foster 2000, 2005). Inkeeping with Boisot, the cognitive process, which lies at the root of the production of the new knowledge required for producing an adaptive response, is distinguished by the cognitive activity of codification and by the cognitive activity of abstraction (Boisot 1995, 1998, Boisot and Li 2006). More precisely, such a cognitive process is distinguished by the cognitive activity of articulation that consists of coordinating the interrelated activities of codification with that of abstraction. In line with Boisot, the complexity of the biophysical and socio-economic environments stands for the amount of data that the cognitive agent has to process so as to produce her own adaptive response. Hence, the cognitive activity of articulation allows the minimization of the amount of data to be processed, and so enables the identification of the data required to produce an adaptive response with the highest degree of efficacy both by assigning data to categories (cognitive activity of codification), and by reducing the number of categories identified to those with the highest degree of efficacy in terms of the goal to be achieved (cognitive activity of abstraction). This line of thinking is defined by Boisot, for the first and only time in a pre-analytical form in his paper dated 2006, with the expression, he coined cognitive production function[21]. In this paper, since Boisot is mainly interested in analyzing the economies of transmission of information and their implications for economics, he only presents a graphic representation of the *cognitive production function* without indicating the analytical expression underlying it. Actually, he uses the *cognitive production function* only in as far as it assits him in achieving the general aim of his paper but not to develop and express it analytically. We intend to develop and make analytically explicit

[21] The expression cognitive production function could bring to mind the evolutionary production function that Boisot had specifically dealt with in his works of 1994, 1998, and 2004, and to the neoclassical production function. In respect to the neoclassical production function, Boisot's evolutionary production function is "*somewhat reminescent of a production function […] but it is not a production function*" (Boisot 2004, p. 59) and moreover, their graphic schemes have only some similarities. With regard to the similarities between the cognitive production function and the evolutionary production function, they can be seen as similar on a graphical level exclusively. In reality, they dipict the relation between different types of variables.

Max Boisot's *cognitive production function* in order to offer a quantitative expression describing in an analytical way the cognitive process lying at the root of the production of new knowledge. As we have summed up in this Section and described in Section 3, the cognitive process allowing the *cognitive agent* to produce new knowledge is distinguished by the cognitive activity of articulation that affects the amount of data to be processed. Therefore, following this line of thinking, the more the cognitive activity of articulation increases, the more the amount of data to be processed decreases, *i.e.* the more data for the highest degree of efficacy can be identified for the optimal adaptive response. As a consequence, on the basis of this inverse ratio we can make analytically explicit the *cognitive production function* as follows:

$$f(x_1, x_2) = \frac{k}{x_1 x_2} \qquad with\ k > 0 \qquad (1)$$

where x_1 and x_2 are the amount of data to be processed, and k the knowledge produced.

Relying on the epistemological foundation defined in Section 3 thanks to the joint analysis of John Foster's and Max Boisot's economic thought, the production of the adaptive response with the highest degree of efficacy may be modelised as a constrained minimum problem. Furthermore, the cognitive agent has to minimise the amount of the available data (data standing for the complexity of the biophysical and socio-economic environments where she interacts and to which she has to adapt) so as to identify only that data required to produce the knowledge for implementing an adaptive response with the highest degree of efficacy in terms of achieving her own aspiration. However, as discussed in Section 3, the minimasation of the amount of data to be processed is constrained by the cognitive activity of articulation of the cognitive agent. In fact, the latter coordinates the interrelated activities of codification with that of abstraction with the purpose of minimizing the amount of available data and distinguishing those data which have the highest degree of efficacy. So we can express the efficacy of the cognitive activity of articulation with a constrained minimum problem:

$$\begin{cases} \min \dfrac{k}{x_1 x_2} \qquad with\ \ k = 1 & (2) \\ a x_1 + \ c x_2 = B & (3) \end{cases}$$

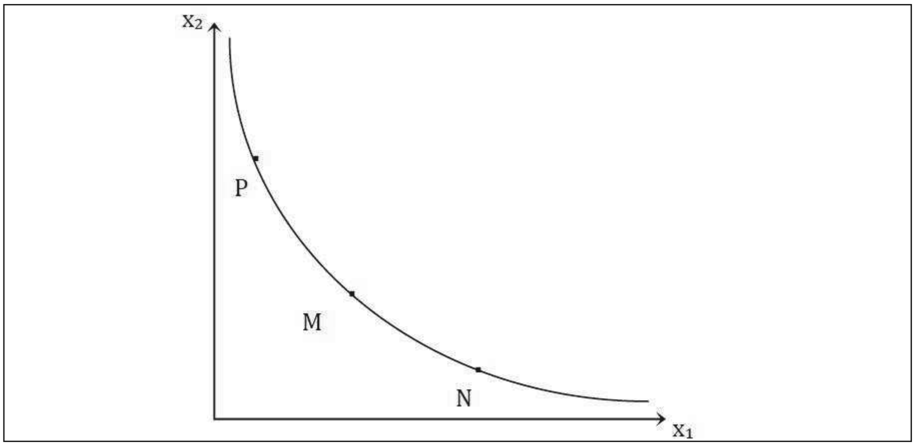
where *a* and *c* are the cognitive activity of abstraction and the cognitive activity of codification of the *cognitive agent*, respectively, and *B* is the total amount of data to be processed. Applying the *Lagrangian* method and calculating the *Hessian* (4), it is possible to find a constrained minimum point (5):

$$\det H_0 = -\frac{2}{x_1 x_2}\left[\left(\frac{a}{x_2} - \frac{c}{x_1}\right)^2 + \frac{ac}{x_1 x_2}\right] < 0 \qquad (4)$$

$$if \quad x_1, x_2 > 0, \qquad a > 0, \qquad c > 0$$

$$M = \left(\frac{B}{2a}, \frac{B}{2c}\right) \qquad (5)$$

Figure 1. Efficacy of cognitive activity in producing knowledge.



Source: our elaboration.

In Figure 1, the minimum point M represents the point at which the *cognitive agent* best uses her own cognitive activity of articulation (please, note that the cognitive activity varies from *cognitive agent* to

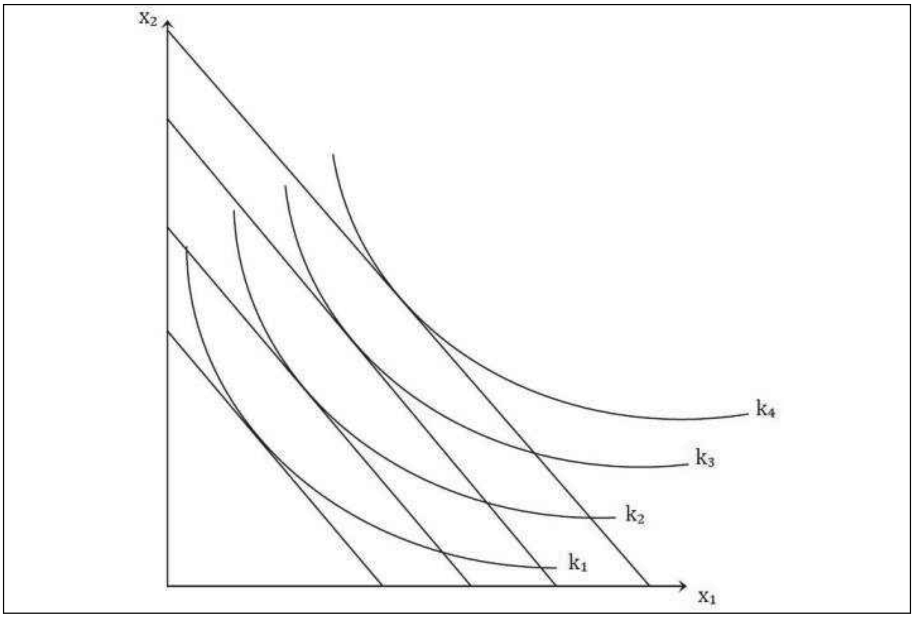
cognitive agent). In reality, each *cognitive agent* has her own cognitive potential for articulation (r), with values ranging from 0 to n_r where $n_r \in \mathbb{R}$, which stands for the maximum cognitive potential for articulation of the *cognitive agent*. Thus, in Figure 1, the minimum point M is the expression of the cognitive agent's maximum cognitive potential *(nr)*.
This minimum point represents the optimal situation for the *cognitive agent* in that, at this point, she is able to minimize the amount of data avaible distinguishing those data with the highest degree of efficacy for the production of knowledge in terms of achieving her aspiration. Thus it follows, that moving away from the minimum point M, for example, to point N or point P, the *cognitive agent* does not make best use of her cognitive activity of articulation, i.e. she uses her own cognitive potential for articulation with values ranging from 0 to n_r-$\varepsilon \ \forall \ \varepsilon > 0$. If we take as an example for our *cognitive agent* the Schumpeterian entrepreneur (Foster 2000, 2005) introduced in Section 3, the minimum point M corresponds to the situation in which the Schumpeterian entrepreneur has distinguished the data with the highest degree of efficacy to produce the knowledge required for optimizing the production process, that is the optimization of the combination of the available factors (human beings and physical object) to achieve her desired level of production.

We now turn to analitically (6) and graphically (Figure 2) represent the possible degrees of knowledge (k) produced by the *cognitive agent* by taking into consideration both her cognitive activity of articulation *(ac)* and the possible scenarios of complexity *(B)* where she may interact. The values of k can be found by satisfying the condition of tangency of (1) with (3):

$$k = \frac{B^2}{4ac} \qquad (6)$$

In Figure 2, the equilateral hyperbolas based upon tangency parameter values (k) represent the knowledge produced by the *cognitive agent*. On the basis of both the epistemological foundation defined in Section 3, and the geometric properties of equilateral hyperbolas, we can conclude that the nearer the vertices are located to the axis origin, the more the *cognitive agent* produces knowledge with a high degree of efficacy in terms of achieving her aspiration. The efficacy of this knowledge produced depends on an increase of the cognitive potential for articulation *(ac)* and/or a decrease in the complexity, i.e. a decrease in the amount of data to be processed *(B)*. The *cognitive agent* can increase her own cognitive potential for articulation by learning and/or by finding herself in situations of decreased complexity thanks to technological innovations, exogenous to the *cognitive agent*[22].

Figure 2. Knowledge produced by the cognitive agent.



Source: our elaboration.

In the specific case of the Schumpeterian entrepreneur (Foster 2000, 2005), described above in Section 3, the efficacy of knowledge by her produced may depend on her learning of specific techniques

22 It is to be noted that technological innovations, exogenous to the cognitive agent, can lead to the creation of new theoretical and material devices which can be used by the cognitive agent to increase the degree of efficacy of her cognitive activity of articulation.

that facilitate her finding optimal combinations of production factors (*e.g. statisticial process control techniques*) and/or on the decrease in the complexity of the biophysical and socio-economic environment in which she interacts, due, for example, to a decrease in the complexity of the bureaucratic system.

5. Summary and concluding thoughts

The interdisciplinary approaches characterizing the current scientific debate on economic theory (Colander 2000a, 2010) attempt to respond to the urge to examine economic phenomena from different perspectives. The same urge that, form the middle of the last century, was felt by those ecomists who took part in the famous Cambridge-Cambridge controversy begun in 1953 by Joan V. Robinson and that was in the same years detected by Herbert A. Simon (1959) and Maurice F. C. Allais (1953) for similar reasons, and, even previuosly, had been dealt with to different degrees, by Thomas R. Malthus (1798), Karl Marx (1867), John M. Keynes (1921, 1936) and Joseph A. Schumpeter (1928, 1934). At the time of the Cambridge-Cambridge controversy, the urge felt by Cambridge UK was to look at the nature of capital factor from a different perspective to that of Cambridge US, not for the mere intellectual pleasure of distinguishing itself in the international scientific world, but in order to respond to the urgent demand to recover the heterogeneous nature of the capital factor for the theory of production. Not only was the importance of recovering this nature opposed to the assumption of Cambridge US that considered exclusively the homogeneity of the capital factor but it was also opposed to the tendency to examine economic phenomena with a methodological approach based on aassumptions and mathematical models which systematically ignore or remove the complexity of reality. This line of thinking has led in time the neoclassical economic thought to develop *ad hoc* theories that, with their presumption of generalizing, force one to view reality from a perspective that oversimplifies that which in effect is by its very nature complex and in continual evolution. This line of thinking is opposed by all the current heterodox economic approaches that attempt to give scientific importance to adopting a scientific perspective that takes into consideration the complexity of reality so as to develop models that interpret and guide it accurately. Such a perspective allows the questioning of logical coherence in the neoclassical mainstream. Anwar Shaikh bringing attention back onto the Cambridge-Cambridge controversy allowed us in Section 2 to describe the impressive example of how meaningful dogma of neoclassical mainstream, *i.e. Cobb-Douglas production function*, can be debunked.

Shaikh succeeds in rendering visible what was invisible to the international scientific world prior to 1974, *i.e.* the *income accounting identity* underlying the *Cobb-Douglas production function.* The debunking of such a dogma and rendering scientifically visible what was not visible beforehand has allowed the further development of the awareness for the urge to rethink economics relying on the complexity of reality as Anwar Shaikh explicitly exhorts in his recent work "*Rethinking Microeconomics*". A consequence of this new found awareness is the discovery that what is essential is the cognitive process that is invisible to the eye. The cognitive process represents, according to the cognitive economics, what is essential to be focused on in that it is the origin and the cause of those phenomena that cognitive economics aims at analyzing. This awareness pushes the urge to overturn the methodological framework of the neoclassical economic research, *i.e.* to move from the deductive method to the inductive one, and to move from the representative agent to the cognitive one. This means focusing attention on the cognitive process with which the individual produces the knowledge required for implementing and optimizing her economic aspirations, choices and lines of action, which are precisely those phenomena that not only does economic science aim to describe but also to explain. Bearing in mind these considerations, in Section 3, we focused on the cognitive process which lies at the root of the production of knowledge required for developing and optimizing the production process, *i.e.* the optimization of the combination of the available factors (human beings and physical object) to achieve the desired level of production (*self-organization*). Relying on the joint analysis of John Foster's and Max Boisot's economic thought, we defined the epistemological foundation applied in Section 4, rendering it possible to develop and make analytically explicit Max Boisot's *cognitive production function* which he defined for the first and only time in a pre-analytical form in his paper dated 2006.

Having exposed a weakness in the neoclassical economic mainstream thanks to *Shaikh's identity*, and having reiterated the essential role of the cognitive process invisible to the eye of those who believe the perfect rationality of the economic agent, led us to jointly analyze John Foster's and Max Boisot's economic thought to define an epistemological foundation rendering it possible to develop and make analytically explicit Max Boisot's *cognitive production function*.

We confide that this epistemological foundation can be further developed and utilised not only to inspire other theoretical reflections on cognitive economics, but also to write scientific protocols for experimental economics with the aim to rethink the economic science responding to Anwar Shaikh's exhortation: "*We therefore need to understand how individual agents actually behave, how they actually react to changes in the macro environment, and to what extent the environment is in turn affected. Behavioral and experimental economics, psychology and sociology, can all have their say.*" (Shaikh 2012, p. 1).

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References

Adler P. S., Cole R. E. (1993), *Designed for learning: a tale of two auto plants*, Sloan Management Review, Massachusetts Institute of Technology, 34 (3), 85-94.

Adler P. S., Cole R. E. (1995), *Designed for learning: a tale of two auto plants*, in Sandberg Å. (ed.), *Enriching production. Perspectives on Volvo's Uddevalla plant as an alternative to lean production*, Avebury, Aldershot.

Agarwal B., Vercellli A. (2005), *Psychology, rationality and economic behavior: challenging standard assumptions*, Palgrave, Basingstoke.

Alberti M. (1938), *La realtà economica*, Rivista Internazionale di Scienze Sociali, Serie III, anno 46, 9(6), 825-848.

Allais M. F. C. (1953), *Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine*, Econometrica, 21(4), 503-546.

Anderson E. (2000), *Beyond homo economicus: new developments in theories of social norms*, Philosophy and Public Affairs, 29(2), 170-200.

Angier N. (2002), *Why we're so nice - We're wired to cooperate*, in New York Times, June 23, 2002. New York.

Aoki M. (2006), *Not more so: some concepts outside the DSGE framework*, in Colander D. (ed.), *Post Walrasian macroeconomics: beyond the Dynamic Stochastic General Equilibrium model*, Cambridge University Press, Cambridge (UK).

Aoki M. (2010), *Corporations in evolving diversity: cognition, governance, and institutions*, Oxford University Press, Oxford (UK).

Ariely D. (2008), *Predictably irrational: the hidden forces that shape our decisions*, Harper Collins Publishers, New York.

Arrow K. J. (1996), *The economics of information: an exposition*, Empirica, 23(2), 120-121.

Boisot M. (1994), *Information, economics, and evolution: what scope for a ménage à trois?*, World Futures: The Journal of General Evolution, 41(4), 227-256.

Boisot M. (1995), *Information space: a framework for learning in organizations, institutions and culture*, Routledge, London.

Boisot M. (1998), *Knowledge assets*, Oxford University Press, Oxford (UK).

Boisot M., Canals A. (2004), *Data, information and knowledge: have we got it right?*, Journal of Evolutionary Economics, 14(1), 43-67.

Boisot M., Li Y. (2006), *Organizational versus market knowledge: from concrete embodiment to abstract representation*, Journal of Bioeconomics, 8(3), 219-251.

Camerer C. (1999), *Behavioral economics: reunifying psychology and economics*, Proceedings of the National Academy of Sciences of the United States of America, 96(19), 10575-10577.

Camerer C., Loewenstein G., Prelec D. (2005), *Neuroeconomics: how neuroscience can inform economics*, Journal of Economic Literature, 43(1), 9-64.

Carter S. (2011), *'On the Cobb-Douglas and all that…': the Solow-Simon correspondence over the aggregate neoclassical production function*, Journal of Post Keynesian Economics, 34(2), 255-274.

Castelfranchi C. (2003), *Per un "programma cognitivo". Rappresentazioni mentali esplicite per l'homo oeconomicus*, Sistemi Intelligenti, XV(2), 247-278.

Cataldo J., Prochno P. (2005), *Cognitive assets: a model to understand the organizational appropriation of collective tacit knowledge,* in Morel-Guimaraes L., Hosni Y. A., Khalil T. M. (eds.), *Management of technology: key success factors for innovation and sustainable development*, Elsevier Ltd, Oxford, 123-134.

Cobb C. W., Douglas P. H. (1928), *A theory of production*, American Economic Review, 18(1), 139-165.

Colander D. (2000a), *The death of neoclassical economics*, Journal of the History of Economic Thought, 22(2), 127-143.

Colander D. (2000b), *Introduction*, in Colander D. (ed.), *Complexity and the history of economic thought*, Routledge, London.

Colander D. (2010), *The economics profession, the financial crisis, and method*, Journal of Economic Methodology, 17(4), 419-427.

Colander D., Holt R., Barkley Rosser J. Jr. (2004), *The changing face of mainstream economics*, Review of Political Economy, 16(4), 485-499.

Conlisk J. (1996), *Why bounded rationality?*, Journal of Economic Literature, 34(2), 669-700.

Corbett A. C. (2002), *Recognizing high-tech opportunities: a learning and cognitive approach*, in Bygrave W. D. et al. (eds.), Frontiers of entrepreneurship research, Wellesley, MA, Babson College, 49-60.

de Saint-Exupéry A. [1943] (1995), *The little prince*, Wordsworth Editions, Chatham; Fr. ed., *Le petit prince*, Gallimard, New York and Paris.

Dosi G., Fagiolo G., Aversi R., Meacci M., Olivetti C. (1999), *Cognitive processes, social adaptation and innovation in consumption patterns: from stylized facts to demand theory*, in Dow S. C., Earl P. E. (eds.), *Economic organization and economic knowledge: essays in honour of Brian J. Loasby*, Edward Elgar, Cheltenham.

Douglas P. H. (1976), *The Cobb-Douglas production function once again: its history, its testing, and some new empirical values*, Journal of Political Economy, 84(5), 903-916.

Eden C., Spender J. C. (1998), *Managerial and organizational cognition*, Sage, London.

Felipe J. (2001), *Aggregate production functions and the measurement of infrastructure productivity: a reassessment*, Eastern Economic Journal, 27(3), 323-344.

Felipe J., Adams F. G. (2005), *'A theory of production'. The estimation of the Cobb-Douglas function: a retrospective view*, Eastern Economic Journal, 31(3), 427-445.

Felipe J., Fisher F. M. (2003), *Aggregation in production functions: what applied economists should know*, Metroeconomica, 54(2), 208-262.

Felipe J., Holz C. (2001), *Why do aggregate production functions work? Fisher's simulations, Shaikh's identity and some new results*, International Review of Applied Economics, 15(3), 261-285.

Felipe J., McCombie J. S. L. (2005), *How sound are the foundations of the aggregate production function?* Eastern Economic Journal, 31(3), 467-488.

Felipe J., McCombie J. S. L. (2009), *On accounting identities, simulation experiments and aggregate production functions: a cautionary tale for (neoclassical) growth theorists*, in Setterfield M. (ed.), Handbook of Alternative Theories of Economic Growth, Edward Elgar, Cheltenham.

Fiol C. M. (2002), *Intraorganizational cognition*, in Baum J. A. (ed.), Companion to organizations, Blackwell, Oxford.

Fisher F. M. (1971), *Aggregate production functions and the explanation of wages: a simulation experiment*, The Review of Economics and Statistics, 53(4), 305-325.

Fisher F. M. (2005), *Aggregate production functions - A pervasive, but unpersuasive, fairytale*, Eastern Economic Journal, 31(3), 489-491.

Foray D. (2000), *L'économie de la cannaissance*, Editions La Découverte, Repères, Paris.

Foster J. (2000), *Competitive selection, self-organization, and Joseph A. Schumpeter*, Journal of Evolutionary Economics, 10(3), 311-328.

Foster J. (2005), *From simplistic to complex systems in economics*, Cambridge Journal of Economics, 29(6), 873-892.

Foster J. (2006), *Why is economics not a complex system science?*, Journal of Economic Issues, 40(4), 1069-1091.

Foster J., Metcalfe J. S. (2001), *Frontiers of evolutionary economics: competition, self-organization and innovation policy*, Edward Elgar, Cheltenham.

Furnam A., Lewis A. (1986), *The economic mind: the social psychology of economic behavior*, St. Martins Press, New York.

Garegnani P. (1960), *Il capitale nelle teorie della distribuzione*, Giuffrè, Milano.

Garegnani P. (1970), *Heterogenous capital, the production function and the theory of distribution*, Review of Economic Studies, 37(3), 407-436.

Ghiselin M., Landa J. T. (2005), *The economics and bioeconomics of folk and scientific classification*, Journal of Bioeconomics, 7(3), 221-238.

Hahn U., Chater N. (1998a), *Understanding similarity: a joint project for psychology*, Case-Based Reasoning and Law, Artificial Intelligence Review, 12(5), 393-427.

Hahn U. and Chater N. (1998b), *Similarity and rules: distinct? exhaustive? empirically distinguishable?* 111-144 in Sloman S. and Rips L. (eds.), *Similarity and symbols in human thinking*, MIT Press, Cambridge (MA).

Harcourt G. C. (1972), *Some Cambrdige controversies in the theory of capital*, Cambridge University Press, Cambridge (UK).

Innocenti A. (2010), *How a psychologist informed economics: the case of Sidney Siegel*, Journal of Economic Psychology, 31(3), 421-434.

Kaldor N. (1956), *Alternative theories of distribution*, Review of Economic Studies, 23(2), 83-100.

Katona G. (1980), *Essays on behavioral economics*, Ann Arbor, University of Michigan Institute for Social Research (MI).

Keynes J. M. [1921] (1948), *A treatise on probability*, second ed. (1948), McMillan, London.

Keynes J. M. (1936), *The general theory of employment, interest and money*, McMillan, London.

Kirzner (1999), *Creativity and/or alertness: a reconsideration of the Schumpeterian entrepreneur*, Review of Austrian Economics, 11(1-2), 5-17.

Kyrö P., Enquist R. (1997), *Management consulting as a developer of SMEs: the changing market structure in this professional service industry on its way towards professionalization*, PKT-säätiö, Helsinki.

Lant T. (2002), *Organizational cognition and interpretation*, in Baum J. A. (ed.), Companion to organizations, Blackwell, Oxford.

Loasby B. (1999), *Knowledge, institutions and evolution in economics*, Routledge, London.

Lundvall B.-Å. (1985), *Product innovation and user-producer interaction*, Aalborg University Press, Aalborg.

Lundvall B.-Å. (1988), *Innovation as an interactive process: fromuser-producer interaction to the national system of innovation*, in Dosi G., Freeman C., Nelson R.R. Silverberg, Soete G. L. (eds.), *Technical Change and Economic Theory*, Printer Publishers Ltd., London, 349-369.

Lundvall B.-Å., Vinding A. L. (2004), *Product innovation and economic theory-user-producer interaction in the learning economy*, in Christensen J. L., Lundvall B.-Å. (eds.), *Product innovation, interactive learning and economic performance*, Elsevier Science, Amsterdam, 101-128.

McCombie J. S. L., Dixon R. (1991), *Estimating technical change in aggregate production functions: a critique*, International Review of Applied Economics, 5(1), 24-46.

Malthus T. R. (1798), *An essay on the principle of population as it affects the future improvement of society*, J. Johnson, London.

Marx K. (1867), *Das kapital*, O. Meissner, Hamburg.

McCombie J. S. L. (2001), *What does the aggregate production function tell us? Second thoughts on Solow's 'Second thoughts on growth theory'*, Journal of Post Keynesian Economics, 23(4), 589-615.

Miller G. (1956), *The magic number seven, plus or minus two: some limits on our capacity for processing information*, Psychological Review 63(2), 81-107.

Nelson R. R., Winter S. G. (1978), *Forces generating and limiting concentration under Schumpeterian competition*, Bell Journal of Economics, 9(2), 524-548.

Nelson R. R., Winter S. G. (1982), *An evolutionary theory of economic change*, Harvard University Press, Cambridge, (MA).

Nietsche F. W. [1883] (1974), *Aphorism 125*, in *Selections from Nietzsche. The gay science* (1882/1887), Vantage Books, New York.

Nonaka I. (1991), *The knowledge-creating company*, Havard Business Review, 69(6), 96-104.

Nonaka I., Takeuchi H. (2001), *Conclusion: social, technical and evolutionary dimensions of knowledge creation*, in Nonaka I., Nishiguchi T. (eds.), *Knowledge emergence. Social, technical and evolutionary dimensions of knowledge creation*, Oxford University Press, Oxford, 286-290.

Nonaka I., Toyama R. (2007), *Why do firms differ? The theory of the knowledge-creating firm*, in Ichijo K., Nonaka I. (eds.), *Knowledge creation* *and management. New challenges for managers*, 13-31, Oxford University Press, Oxford.

Novarese M., Rizzello S. (2004), *Economia sperimentale*, Bruno Mondadori, Milano.

Pasinetti L. L. (1959), *On concepts and measures of changes in productivity*, The Review of Economics and Statistics, 41(1), 270-286.

Pasinetti L. L. (2000), *Critique of the neoclassical theory of growth and distribution*, Banca Nazionale del Lavoro Quarterly Review, 53(215), 187-232.

Patalano R. (2005), *La mente economica. Immagini e comportamenti di mercato*, Laterza, Roma-Bari.

Pedricz W., Gomide F. F. (1998), *An introduction to fuzzy sets*, MIT Press, Cambridge (MA).

Phelps Brown E. H. (1957), *The meaning of the fitted Cobb-Douglas function*, The Quarterly Journal of Economics, 71(4), 546-560.

Pihlaja J. (2005), *Learning in and for production: an activity-theoretical study of the historical development of distributed systems of generalizing*, Department of Education, University of Helsinki, Helsinki University Press.

Pressman S. (2005), *What is wrong with the aggregate production function?*, Eastern Economic Journal, 31(5), 422-425.

Rabin M. (1998), *Psychology and economics*, Journal of Economic Literature, 36(1), 11-46.

Reinert E. (2002), *Schumpeter in the context of two canons of economic thought*, Industry and Innovation, 9(1-2), 23-39.

Reinert H., Reinert E. S. (2006), *Creative destruction in economics: Nietzsche, Sombart, Schumpeter*, in Backhaus J. G., Drechsler W. (eds.), Friedrich W. Nietzsche (1844-1900), *Economy and society, the European heritage in economics and the social sciences*, Vol. III, 55-85.

Robinson J. V. (1953), *The production function and the theory of capital*, Review of Economic Studies, 21(2), 81-106.

Ronstadt R. (1988), *The corridor principle*, Journal of Business Venturing, 3(1), 31-40.

Rullani E. (2004), *Economia della conoscenza. Creatività e valore nel capitalismo delle reti*, Carocci, Roma.

Samuelson P. (1979), *Paul Douglas's measurement of production functions and marginal productivities*, Journal of Political Economy, 87(5), 923-939.

Schumpeter J. A. (1928), *The instability of capitalism*, Economic Journal, 38(151), 361-386.

Schumpeter J. A. (1934), *The theory of economic development*, Oxford University Press, Oxford.

Shaikh A. (1974), *Laws of production and laws of algebra: the Humbug production function*, The Review of Economics and Statistics, 56(1), 115-120.

Shaikh A. (1980), *Laws of production and laws of algebra - Humbug II*, in Nell E. J. (ed.), *Growth, profits and property*, Cambridge University Press, Cambridge (UK), 80-95.

Shaikh A. (2012), *Rethinking microeconomics: a proposed reconstruction*, Working paper No. 06-2012, Department of Economics, New School for Social Research, New York.

Shane S., Venkataraman S. (2000), *The promise of entrepreneurship as a field of research*, Academy of Management Review, 25(1), 217-226.

Shapiro C., Varian H. (1999), *Information rules: a strategic guide to the network economy*, Harvard Business School Press, Boston.

Shepherd D. A., Douglas E. J., Shanley M. (2000), *New venture survival: ignorance, external shocks, and risk reduction strategies*, Journal of Business Venturing, 15(5-6), 393-410.

Shewhart W A. (1931), *Economic control of quality of manufactured product*, Van Nostrand, New York.

Simon H. A. (1956), *Rational choice and the structure of the environment*, Psychological Review, 63(2), 129-138.

Simon H. A. (1959), *Theories of decision-making in economics and behavioral science*, American Economic Review, 49(3), 253-83.

Simon H. A. (1978), *Rationality as a process and as a product of thought*, American Economic Review, 68(2), 1-16.

Simon H. A. (1979), *On parsimonious explanation of production relations*, The Scandinavian Journal of Economics, 81(4), 459-474.

Simon H. A., Levy F. K. (1963), *A note on the Cobb-Douglas function*, Review of Economics Studies, 30(2), 93-94.

Solow R. (1956), *A contribution to the theory of economic growth*, The Quarterly Journal of Economics, 70(1), 65-94.

Solow R. (1974), *Law of production and laws of algebra: the Humbug production function: a comment*, The Review of Economic and Statistics, 56(1), 121.

Solow R. (2000), *The neoclassical theory of growth and distribution*, Banca Nazionale del Lavoro Quarterly Review, 53(215), 149-185.

Sraffa P. (1960), *Production of commodities by means of commodities. Prelude to a critique of economic theory*, Cambridge University Press, Cambridge (UK).

Stiglitz J. E. (2000), *The contributions of the economics of information to Twentieth century economics*, The Quarterly Journal of Economics, 115(4), 1441-1478.

Stiglitz J. E. (2002), *Information and the change in the paradigm in economics*, American Economic Review, 92(3), 460-501.

Stiglitz J. E., Ocampo J. A. , Spiegel S., Ffrench-Davis R., Nayyar D. (2006), *Stability with growth, macroeconomics, liberalization, and development*, Oxford University Press, New York.

Swan T. W. (1956), *Economic growth and capital accumulation*, Economic Record, 32(2), 334-361.

Sylos Labini P. (1995), *Why the interpretation of the Cobb Douglas production function must be radical1y changed?*, Structural Change and Economic Dynamics, 6(4), 485-504.

Sylos Labini P. (2001), *Underdevelopment: a strategy for reform*, Cambridge University Press, Cambridge (UK).

Tapscott D. (1996), *The digital economy. Promise and peril in the age of networked intelligence*, Mc Graw-Hill, New York.

Toates F. (2007), *Biologiacl psychology*, Pearson Edication Ltd, Harlow, UK.

Uhlig H. (2012), *Economics and reality*, Journal of Macroeconomics, 34(1), 29-41.

Vernengo M. (2010), *Conversation or monologue? On advising heterodox economists*, Journal of Post Keynesian Economics, 32(3), 485-499.

Williamson O. E. (1985), *The economic institutions of capitalism*, New York, Free Press.

Williamson O. E. (2002), *The theory of the firm as governance structure: from choice to contract*, Journal of Economic Perspectives, 16(3), 171-195.

Witt U. (2002), *How evolutionary is Schumpeter's theory of economic development?* Industry and Innovation, 9(1-2), 7-22.

Zafirovski M. (2003), *Human rational behavior and economic rationality*, Electronic Journal of Sociology, 7(2), 1-34.

Zuboff S. (1988), *In the age of the smart machine: the future of work and power*, New York, Basic Books.

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AGGREGATE PRODUCTION FUNCTIONS AND THE ACCOUNTING IDENTITY CRITIQUE: CRITICISMS AND MISUNDERSTANDINGS WITH SPECIAL REFERENCE TO TEMPLE

Abstract

The aggregate production function continues to be widely used, notwithstanding the fact that the Cambridge capital theory controversies and aggregation problems have shown that its foundations are tenuous in the extreme. The main reason for its use seems to be the instrumentalist position according to which aggregate production functions "work", in that they usually, but not always, give close statistical fits with plausible estimates of the parameters. However, Anwar Shaikh was one of the first to show that this is simply the result of the existence of an underlying accounting identity together with the use of value data. This interpretation with reference to the subsequent work of Felipe and McCombie has been challenged by Temple in two articles. While Temple concedes the argument has some validity, he erroneously believes, inter alia*, that the critique only holds under certain* ad hoc *assumptions. As a consequence, he argues that the criticism only works "part time". This paper further discusses Temple's arguments and demonstrates that none of them is compelling and the critique, in fact, works "full time". Because of the underlying accounting identity, estimates of the parameters of aggregate production functions cannot be regarded as reflecting the underlying technology of the industry or economy and Anwar Shaikh's original insights remain valid.*

JEL Classification: C43, O11, O16, O47, O53.

Keywords: Accounting Identity, Aggregate Production Function, Aggregation Problems, Value-Added Identity, Value Data.

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> "I am informed ... that attempts to explain the impossibility of using aggregate production functions in practice are often met with great hostility, even outright anger. To that I say ... that the moral is: 'Don't interfere with fairytales if you want live happily ever after'." (Franklin Fisher, 2005, p.491).

> "[Neoclassical economists] don't like the true and simple; they like fairy tales and humbug." (With apologies to Edmond de Goncourt, 1822-1896).

1. Introduction

It is nearly ninety years ago since Cobb and Douglas (1928) first published the results of their estimations of the now familiar multiplicative aggregate production function that bears their name. In that paper, they also set out the conditions for the aggregate marginal productivity theory of factor pricing. Douglas (1976) subsequently argued that the close approximation that the estimated output elasticities bore to their respective factor shares confirmed the assumption of perfectively competitive markets and disproved the Marxian explanation of the distribution of income. It is sometimes forgotten now that Cobb and Douglas's original study was so heavily criticized by the econometricians and Douglas's *a priorist* colleagues at Chicago that he nearly gave up the whole endeavor (Douglas, 1967). Nevertheless, he persevered with his colleagues and felt eventually vindicated by being elected President of the American Economic Association in 1947. The title of his Presidential address, "Are there Laws of Production?" (Douglas, 1948), was largely rhetorical. The ultimate accolade came in 2011 when the article Cobb and Douglas (1928) was nominated as one of the 20 most "admirable and important articles" published in the *American Economic Review* in the last hundred years (Arrow, 2011). By implication, all doubts about the validity of the neoclassical one-sector aggregate production function and the associated aggregate marginal productivity conditions as a meaningful representation of the technical conditions of production of individual industries, or the whole economy, had been banished. The unanswered serious reservations about aggregation problems (Fisher, 1992, 2005) or the Cambridge capital theory controversies (Harcourt, 1972, Birner, 2002, and Cohen and Harcourt, 2003) have disappeared from the collective memory of mainstream economists or been relegated to the history of economic thought.[1]

While the Cambridge capital theory controversies have attracted more notoriety, the more general aggregation problems have proved even more damaging. Franklin Fisher (2005, p.490), who has probably done most work on this topic was forced to conclude that "even under constant returns, the conditions for aggregation are so very stringent as to make the existence of aggregate production functions in real economies a non-event. … *One cannot escape the force of these arguments by arguing that aggregate production functions are only approximations*." (Emphasis added.)

It was half a century ago that Samuelson (1962) tried, and failed, to provide a theoretical foundation for the use of the aggregate production function as a "parable" or useful approximation reflecting the myriad underlying complex micro-production function relations. Samuelson attempted to show that a series of two-sector production techniques (a pair of fixed-coefficient techniques to produce a consumer and a capital good) could give rise to an aggregate relationship with all the standard properties of, say, an aggregate Cobb-Douglas production function. While problems of the measurement of capital and more general aggregation problems can be traced back to the Classical economists and had been repeatedly emphasized in the 1950s by Joan Robinson (1953-54, 1956), this paper galvanized a debate in capital theory between Cambridge, Massachusetts and Cambridge, UK.[2]

The outcome was that unless the capital-labour ratios of the two sectors were identical, the phenomenon of reswitching could occur. A technique could be the most profitable at both a high rate and a low rate of interest, but not at an intervening rate of interest. There was no necessary inverse monotonic relationship between the rate of profit and the capital-labour ratio, as implied by the standard aggregate neoclassical production function. For reswitching to be ruled out

[1] It is indicative that Cohen and Harcourt felt compelled to write a reminder for the economics profession in the 2003 issue of the *Journal of Economic Perspectives* in the "Retrospectives" section entitled "Whatever Happened to the Cambridge Capital Theory Controversies?" and that Birner's 2002 volume, *The Cambridge Controversies in Capital Theory*, is part of the *Routledge Studies in the History of Economics*.

[2] Strictly speaking, it was the erroneous attempt of Lehavari (1965) to remove limiting conditions in Samuelson's paper that crystallized the debate.

meant, in effect, any technique in Samuelson's model had to have identical capital-labour ratios. This effectively meant Samuelson had not escaped from assuming that there was a one-sector model. This phenomenon of reswitching had been known for some time, being regarded by Joan Robinson as merely the "Ruth Cohen curiosum" and had been discussed by Champernowne (1953-54) and Sraffa (1960), but assumed greater prominence as a result of Samuelson's (1962) paper. Indeed, Samuelson's (1966) summing up of the debate gave a simple example as to why reswitching occurs. The ensuing debate was published as a symposium in the November issue of the 1966 *Quarterly Journal of Economics.* For a time, the outcome was seen as the death knell for the aggregate production function. In the immediate aftermath, advanced undergraduate textbooks, especially on growth theory, carried discussions of the debate (Wan, 1971, Jones, 1975, Haache, 1979). But, surprisingly, by the early 1990s all references to it had disappeared (Barro and Sala-i-Martin, 1995, Aghion and Howitt, 2009, Weil, 2005, Acemoglu, 2009).

Given that it was a debate involving formal logic, rather than the plausibility about certain assumptions (such as the relevance of the assumption of perfect competition), the reason for this collective amnesia remains an important question for the history of the subject. In this paper, we focus on what we see as the most convincing reason, namely the instrumentalist defence (Friedman, 1953) that, not withstanding the aggregation problems and the implications of the Cambridge capital theory controversies, the aggregate production function "works". In other words, estimations of the aggregate production function often give close statistical fits with "plausible" estimates of the parameters.[3]

However, this methodological stance was demolished in important articles by Phelps Brown (1957), Simon and Levy (1963) (both for estimation using cross-section data) and especially the influential papers of Anwar Shaikh (1974, 1980).

[3] Han and Schefold (2006) use input-output tables for the OECD countries to test empirically the possibility of reswitching. They find that the existence of at least three switch points between two wage curves is negligible; it occurs only 0.73% of the time. However, the input–output coefficients used are *value* measures, not physical technical coefficients, and so it is perhaps not surprising that these results come to the same conclusion as the direct estimation of the aggregate productions and are subject to the same critique. (See Gandolfo (2008) for a more general critique of this approach.)

Shaikh (1974) made two separate critiques of Solow's (1957) famous paper on the aggregate production function and technical change. First, he showed that Solow's method of 'deflating' the time-series data to remove the influence of the growth of technology when estimating the production function reduced the latter procedure to a tautology, as Solow (1974) conceded. To drive the point home, Shaikh constructed a data set where the observations for the capital-labour ratio spelt out the word HUMBUG and this likewise gave a near perfect fit to the Cobb-Douglas production when Solow's estimation method was used. Secondly, and more importantly, Shaikh showed how the underlying accounting identity would ensure a near perfect statistical fit to the aggregate production, even though the latter did not exist. Shaikh (1980) is a further elaboration of this argument and also a devastating refutation of Solow's (1974) critique of Shaikh (1974). Mention must also be made of Shaikh's excellent simulation paper that showed how, because of the accounting identity, data generated by Goodwin's (1967) fixed-coefficient growth model will also give a good statistical fit to the Cobb-Douglas 'production function', because of the stability of the factor shares.

The baton was then picked up by the present authors who have extended and generalized the critique and shown how the results of some seminal papers and models are dependent upon the accounting identity. These include the augmented Solow model of Mankiw *et al.*, (1992) (Felipe and McCombie 2005a); Hall's (1988) estimates of the mark-up (Felipe and McCombie, 2002), and labour demand functions (Felipe and McCombie, 2009).

Surprisingly, this critique has been almost totally ignored by both post-Keynesians and mainstream economists alike. There is no entry in the *Elgar Companion to Post Keynesian Economics* and literally two sentences in King's (2002) history of post-Keynesian economics. This is not withstanding Joan Robinson's comment (1970, p. 317, omitting a footnote) that the "statistical defence" of aggregate production function "must have needed an even tougher hide to survive Phelps Brown's article on "The Meaning of the Fitted Cobb-Douglas Function" than to ward off Cambridge Criticism of the marginal productivity theory of distribution".

An exception is Temple (2006, 2010), who in two articles commented specifically on the accounting identity critique and authors' work in this area. While conceding some points, he argues that the critique only "works part-time" Temple (2010). This was in spite of a rebuttal to Temple (2006) by Felipe and McCombie (2010a), which Temple largely ignored. Temple's papers are important if only to the extent that they demonstrate what may be common fundamental misunderstandings of the critique by mainstream economists and hence the neglect of its damaging implications. Consequently, in this paper we return to this debate and show that the critique works "full time", *pace* Temple. Felipe and McCombie (2012) focus on some broader methodological issues concerned with the critique.

2. The instrumental defence of the aggregate production function

A common defence of the use of unrealistic assumptions is Friedman's (1953) methodological stance that a "theory is to be judged by its predictive power of the class of phenomena which it is intended to 'explain' … the only relevant test of the *validity* of a hypothesis is comparison of its predictions with experience" (pp. 8-9). The realism of its assumptions is irrelevant. The only problem is "whether they are sufficiently good approximations for the purpose at hand. And this question can only be answered by seeing whether a theory works, that is, whether it yields sufficiently accurate predictions" (p. 15). While Friedman's methodological stance has been heavily criticized (Samuelson, 1963, Kincaid, 1996, pp. 227-228), his approach is still widely accepted by economists.

As far as the aggregate production function is concerned, Wan (1971, p.71) views it as an empirical law in its own right, which is capable of statistical refutation. The instrumental defence is also implicit in Solow's remark to Fisher, that "had Douglas found labor's share to be 25 percent and capital's 75 per cent instead of the other way around, we would not now be discussing aggregate production function" (cited by Fisher, 1971, p.305). Ferguson (1969, p.xvii) explicitly made this instrumental defence with respect to the criticism about the measurement of capital as a single index in Cambridge capital theory controversies. "Its validity is unquestionable, *but its importance is an empirical or an econometric matter* that depends upon the amount of substitution there is the system. Until the econometricians have the answer for us, placing reliance upon [aggregate] neoclassical economic theory is a matter of faith. I personally have faith".

The citation in the introduction to this paper as to why the Cobb-Douglas (1928) article should be regarded as one of the twenty most influential articles in the last hundred years published in the *American Economic Review* states: "Cobb and Douglas explored the elementary properties and implications of the functional form, and pointed to the approximate constancy of the relative shares of labor and capital in total income as the *validating empirical fact* (Arrow *et al*., 2011, p.2). Hoover (2012, p. 326) also adopts an instrumental position in his intermediate macroeconomic textbook.[4] He briefly notes the aggregation problems which "are well beyond the scope of this book". So instead, "our strategy will be to start with a conjecture that the economy can be described by a particular production function [the Cobb-Douglas], one that shares important properties with microeconomic production functions. We will then test our conjecture empirically. *If it seems to describe the data well, we shall be satisfied that it provides a useful approximation*" (emphasis added)." What is this test? It is simply the approximate constancy of shares and "provides a good reason to take the Cobb-Douglas production function as a reasonable approximation of aggregate supply in the U.S. economy" (p. 330).

The problem with this line of reasoning is that a production function is theoretically a relationship between *physical* units of homogeneous output (say, widgets) and the flow of physical inputs, for example, efficiency-adjusted labour services and the flow of services of homogeneous physical capital inputs. [5] However, in practice, the use of constant-price *value* data has to be used for both output and capital. The existence of an underlying accounting identity that definitionally relates these to variables means that any regression of an "aggregate

[4] As Kuhn (1970) points out, textbooks are crucial in that they are generally taken by students as being correct and they set the legitimate methods and assumptions for "puzzle solving" within the paradigm.

[5] We leave aside the problems as how to measure these flows. In practice, the levels of employment and the capital stock are used, sometimes adjusted for changes in capacity utilisation.

production function" will simply be an estimate of a mathematical transformation of this accounting identity. Hence, with a little ingenuity it should be always possible to obtain a perfect, or at least, a very close statistical fit. This argument can be made a number of ways, and so we will only give the case with respect to time-series (Felipe and McCombie, 2005b). The definition of value added is given by the accounting identity:

$$V_t \equiv W_t + \Pi_t \equiv w_t L_t + r_t J_t \tag{1}$$

where V is constant price value added, W is the total wage bill, Π is total profits, w is the average wage rate, L is the employment, r is the *ex post* or earned rate of profit and J is the constant price value of the capital stock, usually calculated by the perpetual inventory method (the argument also holds for gross output where the value of output also includes the cost of materials). Expressing equation (1) in growth rate form gives:

$$\hat{V}_t \equiv a_t \hat{w}_t + (1 - a_t)\hat{r}_t + a_t \hat{L}_t + (1 - a_t)\hat{J}_t$$

$$\equiv \lambda_t + a_t \hat{L}_t + (1 - a_t)\hat{J}_t \tag{2}$$

which is compatible with *any* state of competition and whether or not an aggregate production function actually exists.
A general form for the aggregate production function is $V = f(L, K, t)$ which, expressed in growth rates, gives:

$$\hat{V}_t = \lambda_t + \alpha_t \hat{L}_t + (1 - \alpha_t)\hat{J}_t \tag{3}$$

If the usual neoclassical assumptions hold, including that factors are paid their marginal products, equation (3) can be written as:

$$\hat{V}_t \equiv \lambda_t + a_t \hat{L}_t + (1 - a_t)\hat{J}_t \tag{4}$$

which is formally equivalent to the accounting identity, and where the rate of technical progress or, strictly speaking, the growth of total factor productivity is given from the dual by $\lambda_t = a\hat{w}_t + (1 - a_t)\hat{r}_t$.

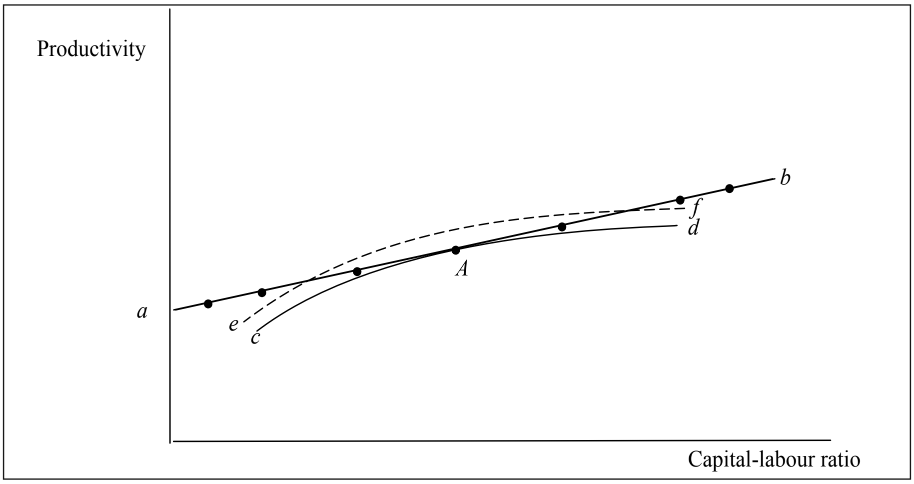
Neoclassical production theory generally estimates a specific form of equation (3), often assuming that the rate of technical progress is a constant with a random error.

3. A common misunderstanding of the critique

The critique has often been dismissed out of hand by some neoclassical economists as simply an elementary failure to understand the optimization conditions underlying producer equilibrium. A firm will produce the optimal volume (and we use this word advisedly) of output where the cost curve is tangential to the highest isoquant. A corollary is that if we write the production function in intensive form $Q/L = f(K/L)$, then the equilibrium level of output will be given by Euler's theorem where the cost function expressed in the form $Q/L = f_L + f_K K/L$ is tangent to the production function. In this specification Q and K are homogeneous physical units (numbers of widgets and units of leets respectively). f_L and f_K are the marginal products expressed in physical units. This may simply be transformed into monetary terms by expressing the cost function as $pQ/L = f_L + pf_K K/L = w + \rho K/L$, where p is the price (in £s), w is the wage rate and ρ is the rental price of capital, again both measured in £s. Consequently, $pQ/L = w + \rho K/L$ is also an identity as the total factor payments must exactly equal the value of output. Thus, in terms, of figure 1, this occurs at point A where the cost equation and the production function (in intensive form) are at a tangent. All, for example, it is (erroneously) argued that Simon and Levy (1962) accomplished with their Taylor series expansion of the cost and the Cobb-Douglas production function is to prove that at the point of tangency (or strictly speaking in the neighbourhood of A), the optimal level of output per worker is given by two equivalent equations. This, so the argument continues, does not imply that there is no underlying production function.

Unfortunately, this argument is a classic *petitio principii* or a case of 'begging the question'. This is the fallacy of assuming in the premise of an argument (namely, the existence of an aggregate production function) that which one wishes to prove in the conclusion (namely, the existence of aggregate production function).

Figure 1. The Cobb-Douglas approximation to the linear accounting identity



Source: our elaboration.

Although the analysis above has been conducted in terms of value terms, as the price (in, say, £'s) is known, the aggregate production function can simply be written in physical terms and apart from the usual econometric specification problems, this can be estimated and reflects the underlying technology of the economy. This implicitly assumes that $PV = V \equiv pQ$ where P is the price deflator (normalised for convenience to unity) and V is value added. Hence the use of value data (value added and the constant price value of the capital stock) can be used as proxies in the estimating of an aggregate production function. Hence, it can be inferred from the findings that, say, a Cobb-Douglas gives a good fit to the data implies that the aggregate elasticity of substation is a meaningful technological concept and takes a value of unity.

A moment's reflection will show there is a serious error in this argument.

Value added is actually defined as $V_{t(0)} \equiv \sum_i p_{i0}Q_{it}$. In other words, value added at time *t* in base-year *0* prices equals the sum of all the physical units of output (chairs, personal computers, aircraft, etc.) multiplied by their unit prices. The total value of the capital stock is calculated by the perpetual inventory method using the cumulative deflated *values* of the investment goods, minus an assumed rate of depreciation of the value of the existing capital stock. Hence, we know that the linear accounting identity must hold be definition and the direction of 'causation' is directly from this to the form of the aggregate production function which in all probability does not exist. This is not to say that there is no relationship at the micro-level between quantities of inputs and outputs measured in *physical* terms; obviously there is a relationship between output and the inputs. However, the form of even these micro-production functions cannot be recovered from the data unless physical data are used and even then the relationship may be very complex, obscured by differences in x-efficiency, and change over time, etc.

Hence, in terms of figure 1, if we have several observations for the accounting identity (and for expositional purposes we assume that the wage rate and the profit rate is constant), and fit, say, a Cobb-Douglas production function, then the best statistical fit will be given by the dotted line *ef* even though no aggregate production function exists. Simon (1979) has shown that the degree of error induced by fitting a Cobb-Douglas production function to the linear accounting identity in these circumstances for plausible ranges in the capital-labour ratio is very small, around 5%. Simon and Levy (1963, p. 94) are quite clear about what they see as the chain of reasoning: "Thus, the existence of a fitted Cobb-Douglas function with a value of k [the output elasticity of labour] in agreement with the actual k [labour's share in output] does not imply that the underlying production function is truly Cobb Douglas. In fact, we expect this agreement when the true function is given by [the accounting identity]."

The fact that distinction between value added data and the value of homogeneous output is not explicitly made accounts for this above confusion over the argument.

4. Temple's (2006, 2010) criticisms of the accounting identity critique[6]

Temple (2006) has raised a number of objections to our arguments (the latter have been brought together in the book by Felipe and McCombie (2013)) concerning the accounting identity, none of which is compelling (Felipe and McCombie, 2010a). In a rejoinder, Temple (2010) concludes that we make substantially more of the criticism of production functions estimated using value data (at any level of aggregation) than it deserves. Hence, he labels it "the part-time tyranny of the identity". However, he largely repeats the objections of his 2006 paper and generally ignores our counter-arguments. While he concedes that there are some areas of agreement and that the argument deserves to be better known, he contends that we have gone too far and exaggerated the scope and implications of the argument. To do so, he reviews some of our work and disputes a number of our conclusions.

We stand by the full extent of the implication of the argument, namely, that the use of value data (as opposed to physical quantities) in the estimation of *any* specification of an aggregate production function, whether or not it is a Cobb-Douglas, precludes the researcher from interpreting the regression results as the technological parameters (e.g., the factor output elasticities or the elasticity of substitution). However, Temple persists in maintaining that the critique *only* relates to the Cobb-Douglas relationship not withstanding the above argument. We also argue that it is true for any level of aggregation using value data. The aggregate production function is, in fact, unlikely to exist, not least because of the serious aggregation problems and variations in *x*-efficiency, etc. The only certainty is that the regression results and the values of the estimated parameters are determined by the accounting identity. The tyranny of the identity works "full time".

Temple, nevertheless, agrees with us on two points. The first one is that the aggregation problem should receive more attention in the literature than it does, although he argues that there are other approaches that are not so reliant on aggregation, e.g., the use of multi-sector models, reduced-form regressions, and methods to infer productivity

[6] This is taken from Felipe and McCombie (2013, chapter 12).

levels from bilateral trade data (Temple 2010, 686). We do not deal with this view here, but concern ourselves solely with the problems posed by the accounting identity for the aggregate production function.

As we have noted above, both the Cambridge capital controversies and the more general aggregation literature suggest that the aggregate production function does not exist. "Indeed, Temple (2005, 438) himself gives the simple example that two Cobb-Douglas production functions with different exponents cannot be aggregated to give a single Cobb-Douglas production function.[7] Surprisingly, later in his comment, he declares himself "agnostic" on this issue (Temple, 2010, 689), although no compelling reasons are given for this. It is not clear why aggregation problems disappear and a true production function can be estimated if "we have no prior reason to believe that output and inputs are badly measured" (Temple, 2010, 689), especially when it is agreed that the accounting identity critique has nothing to do with measurement errors.

The second area where there is agreement is that an applied researcher may appear to obtain meaningful results from estimating a production relationship, even when the researcher is making assumptions that do not hold in the data. "One important instance arises when factors are not paid their marginal products. In that case, although researchers often interpret their results as if the estimated parameters can be used to derive output elasticities, the identity *suggests* that the estimates may be more closely related to the factor shares" (Temple 2010, 686; emphasis added). This would seem to go a long way to conceding our position and poses difficulties for understanding the rationale for his criticisms. We would indeed agree with this statement, except that the identity *shows*, not *suggests*, that the estimated coefficients will take values that are equal to the factor shares, even when no well-defined aggregate production function exists.[8] An implication of Temple's

[7] Nevertheless, ironically, this does not stop Temple (2005) from assuming that the production functions of agriculture and non-agriculture are each represented by an aggregate Cobb-Douglas, and that factors are paid their aggregate marginal products, as if aggregation problems did not matter for these sectors.

[8] Where this does not prove to be the case, it is because the mathematical transformation of the identity to give a specific functional form (i.e., what the neoclassical economist calls the aggregate production function) does not accurately mirror the identity. In other words, this is when the statistical fit of the transformation is less than perfect. We elaborate on the reasons for this below.

statement cited above is that given that the researcher has only access to constant-price value data; it can never be known whether or not the researcher is correctly estimating the parameters of a production function, or, indeed, whether or not the latter exists. This is precisely our critique.

Yet, at times in his reply, Temple (2010) takes the opposite view, and argues erroneously that if factor shares vary to an unspecified extent and the researcher can correctly specify "total factor productivity", all will be well. The aggregate production function can then be estimated and the values of the coefficients will correctly reflect the aggregate technological parameters of the economy, albeit with the necessity of finding the most appropriate statistical estimating technique. (We dealt with the erroneous arguments of Temple (2006) in Felipe and McCombie (2010a).

In the rest of this article, we briefly point out the problems with Temple's (2010) arguments. Broadly speaking, there are two issues that we wish to emphasize. First, Temple erroneously continues to imply that the critique only holds if certain *ad hoc*, or what he terms auxiliary, assumptions are made; typically the "stylized facts" that factor shares are constant and the weighted growth of the wage and profit rates are constant. To this, he incorrectly adds, in the case of our critique of Mankiw *et al.*'s (1992) growth model that we have, of necessity, to assume also a constant capital-output ratio for the criticism to hold. At times, as we noted above, he seems to assume that the critique only applies to the case of the Cobb-Douglas and so, presumably, once there is some variability in factor shares, he implies that we can actually be confident we are estimating a "true" aggregate production function. He nowhere answers the question posed by Felipe and McCombie (2010a) as to how much variability in factor shares is required to suddenly remove the problems posed by the accounting identity and aggregation problems.

Secondly, we show below that his argument at times reduces to *petitio principii*, or circular reasoning. Temple sometimes *assumes* that the aggregate production function exists, and uses this assumption to supposedly counter the argument that the relationship between output and inputs in value terms does not reflect a technological production relationship.

5. On some misunderstandings and misinterpretations of the accounting identity critique

Economic rents and the actual and virtual accounting identities.

Temple implies that for the critique to hold, (his discussion is in the context of the Solow residual), rents need to be excluded from the accounting identity (Temple 2010, 688). However, the accounting identity simply shows how value added is measured. As we have seen, this is given by the identity $V \equiv W + \Pi$, where W is the wage bill and Π is the operating surplus. The latter includes all types of profits. All we do is to split the wage bill into the product of the average wage rate (w) multiplied by employment (L); and the surplus into the product of the average rate of return (r) times the value of the stock of capital (J). This implies that $V \equiv W + \Pi = wL + rJ$. This requires no economic assumptions whatsoever and holds true by definition.

Unfortunately, Temple seems to confuse the "actual" accounting identity with what we have termed elsewhere the neoclassical "virtual" identity (Felipe and McCombie, 2007), based upon the assumption of perfectly competitive markets and optimisation. Consequently the latter may, or may not, hold in reality.

This is usually derived at the microeconomic level by applying Euler's theorem to the micro production function, together with the assumption that the marginal theory of factor pricing holds, i.e., $pY = p\,F(L, K) = p(\partial Y/\partial L)L + p(\partial Y/\partial K)K = wL + \rho K$ where Y is output and K is capital, both measured in *physical* terms, and r is the rental price of capital. p is the price of a unit of output. This chain of reasoning is then applied seamlessly in neoclassical production theory to the macroeconomic level, regardless of the fact that output is value added (V) - not units of physical output - and capital is not the stock of homogeneous structures and equipment, but the constant-price value of the stock of capital (J).[9] If an estimate of the competitive rate of profit is used in the accounting identity, then the

[9] The argument also holds for gross output by adding intermediate materials.

implied economic rents would have to be deducted from value added and the argument follows through exactly (Felipe and McCombie, 2007).

The accounting identity critique does not hold only when factor shares are constant.

Temple argues to the contrary that the identity argument only holds with constant factor shares and a constant weighted growth of the wage rate and the rate of profit, which he claims we have to introduce as arbitrary or *ad hoc* assumptions. This is most clearly seen in the following:

> All of their [Felipe and McCombie's] arguments share a common structure which is to manipulate the value added identity, add some auxiliary assumptions, and then show – under these maintained assumptions - that the data will appear to have been generated by a production relationship of a certain type, typically, *but not always*, Cobb-Douglas, even when no such relationship exists. (Emphasis added.)
>
> Here is their main claim stated explicitly: "Can a researcher using value data ever establish whether or not the coefficients reflect a production function, or are the simply predetermined by the value added identity? Our answer is that unequivocally the results are always determined by the identity". *Yet, the very next sentence in their paper assumes that the weighted average of growth of factor prices and factor shares are all constant.* These assumptions are needed to show how the identity leads to estimates that appear to support a Cobb-Douglas production function. Since these assumptions will not always be met, it is clear that the value added identity does not always lead to a spurious Cobb-Douglas result (Temple, 2010, 687-688, emphasis added).

Temple's argument, consequently, is that we assert (correctly in our opinion) that the results are "always determined by the identity", even when these assumptions are not met. But Temple disputes this and argues that if a good proxy for total factor productivity (TFP) can be found "there is no reason why a researcher should not discriminate between, say, a translog and a Cobb-Douglas specification. Say that the data have been generated by a stable production relationship, and the researcher specifies this relationship correctly, including controls for productivity differences such as TFP. In that case, the researcher is estimating a model that corresponds to the data generating process. There is no reason for the estimates to be biased, or for the parameters to be unidentified. In contrast, and for the same reason – the equivalence between the form of the estimated model and the generating process – the dynamic version of the value added identity cannot do better than this. It will certainly do worse, when the auxiliary assumptions introduced by Felipe and McCombie are not a good approximation to the data" (Temple 2010, 688, omitting a footnote).

The circular reasoning of Temple is readily apparent here. He assumes that a "stable production function exists" [i.e., the data is generated by an aggregate production function]. This can be estimated provided TFP can be correctly specified, another concept dependent on the aggregate production function. Of course, if one adopts this *petitio principii* then the problem is merely one of determining the best specification and estimation techniques, which has been the subject of the numerous articles that have estimated the production function. As the "dynamic value added identity" is an identity, then a better way of putting it is that the estimate of any specification of an "aggregate production function" can do no better than this, rather than *vice versa* as Temple argues in the quotation cited above. And if factor shares vary, then, of course, the functional form that gives the best fit to the identity will not be the Cobb-Douglas. But this ignores (rather than refutes) the criticism that what is driving the results is the identity as the estimates are not of a behavioural equation. We spelt this out in Felipe and McCombie (2010a) immediately prior to our argument in the above citation and ignored by Temple (2010).

Consequently, the argument follows through *whether or not* factor shares and the weighted growth of the wage rate and the rate of profit are constant. In practice, as we have noted, researchers will attempt to find an explicit functional form that will give a good fit to the data generated by equation (1). Thus we have:

$$V_t \equiv w_t L_t + r_t J_t \quad \Rightarrow \quad \hat{V} \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t + a_t \hat{L}_t + (1-a_t)\hat{J}_t$$

$$\Rightarrow \quad \hat{V}_t \equiv \lambda_t + \alpha_t \hat{L}_t + \beta_t \hat{J}_t \Rightarrow V_t = f(L_t, J_t, t) \tag{6}$$

with the arrows showing the direction of causation.

$(1\text{-}\alpha_t)$.

This implies that $a_t \equiv \alpha_t$ and $(1\text{-}a_t) \equiv \beta_t \equiv (1\text{-}\alpha_t)$. As we have noted, economists try to find a specific mathematical functional form that will closely fit the data generated equation (3) and hence, by implication, the underlying identity. If, and only if, the weighted average of the growth of the wage and profit rates is constant, and factor shares are also constant, will a conventional Cobb-Douglas relationship fit this criterion. If they are not constant, then a more flexible functional form that contains the Cobb-Douglas as a special case, such as a Box-Cox transformation, or the translog, will be required. But these mathematical isomorphisms should not be regarded as aggregate production functions. Consequently, the argument does not apply solely to the case where the aforementioned assumptions hold. As this has been quite generally emphasized throughout the literature on the subject, and especially in Felipe and McCombie (2010a), it is surprising that Temple should think otherwise.

The argument is consequently a matter of methodology and logic. What we show are the conditions under which a given form of the production function, say the Cobb-Douglas, would yield good results in terms of the usual statistical diagnostics. This is very different from claiming that specific assumptions or some structure must be imposed for the critique to hold. In fact, an implication of the accounting critique is that unless factor shares and $a_t\hat{w}_t + (1 - a_t)\hat{r}_t$ are approximately constant, the estimation of the equation $V = A_o e^{\lambda t} L_t^{\alpha} J_t^{\beta}$ using time-series data will be likely to yield poor results. That is to say, implausible estimates of the factor elasticities that are very different from the values of the factor shares and may even be negative. The identity shows that a better fit can be obtained by both a more flexible functional form and time trend. But if these stylized facts hold, then the goodness of fit will potentially be unity and the estimated elasticities must equal the factor shares.

Given this, why has the Cobb-Douglas proved so durable, and why does it so often give a good statistical fit to the data? If we integrate equation (2), we obtain $V \equiv a^{-a}(1-a)^{(1-a)} w^{a} r^{(1-a)} L^{a} J^{(1-a)}$. This is not an approximation, but an isomorphism: it holds *exactly* for any particular year, whether it is for, say, the UK economy or an individual firm. As it is a stylized fact (not an assumption) that factor shares do not change greatly between firms in the same industry, and wages and profits show little variation compared to L and J, estimating cross-section production functions gives a good fit with a surprisingly high R^2 of over 0.9, and the estimated output elasticities equal the factor shares (Douglas, 1976). Time-series data often yield worse results, not because factor shares change dramatically over time (they do not), but because $a\ln w_t + (1-a)\ln r_t$ is often not well approximated by a linear time trend, or $a\hat{w}_t + (1-a)\hat{r}_t$ by a constant, as they are subject to cyclical fluctuations.

Temple also discusses the distinction between the Solow residual and TFP growth. To interpret the Solow residual as a measure of TFP growth requires equality between factor prices and marginal products. What we demonstrate is that the data will always show this, provided one uses the right functional form. This section of Temple's argument also borders on the *petitio principii*. As is well-known, Solow (1957) came to the "startling" result (Solow, 1988, 313) that the growth of factor inputs for the US explained less than one eighth of the growth of labour productivity, while the rate of technical progress (which is how Solow loosely interpreted the residual) explained the remaining seven-eighths. Far from being startling, a back-of-the-envelope calculation with the identity shows that this result is inevitable. The growth of total factor productivity is defined as $T\hat{F}P \equiv a\hat{w} + (1-a)\hat{r}$. The neoclassical assumptions are the existence of an aggregate production function and that factor shares are equal to the aggregate output elasticities. If factor shares are roughly constant (with a labour share of about 0.75) and the rate of profit does not vary systematically over time, by using value data the growth of TFP will equate to 75% of the rate of growth of productivity.[10] In fact, Solow found the proportion slightly larger than this, because the rate of profit declined over the period under consideration.

Temple agrees that the growth accounting approach requires the existence of a well-behaved production relationship, but argues that a more general approach would be needed where the equality between marginal products and factor products does not hold. As examples, he cites the work of Basu, Fernald and Kimball (2006), Fernald and

[10] Note that in these circumstances $T\hat{F}P = a\hat{w} = a(\hat{V} - \hat{L})$ where $a \approx 0.75$. The growth accounting approach normally calculates TFP growth over several years and given that factor shares are not constant, uses the average value of the shares (based on the Tornqvist approximation to the Divisia index).

Neiman (2010) and Temple and Wöbmann, (2006). It is difficult to see any relevance of these articles to the present debate because all commence by explicitly *assuming* an aggregate production function exists and use value data. Fernald and Neiman (2010) actually specify a Cobb-Douglas production function!

The Mankiw, Romer and Weil (1992) growth model is merely a misspecified identity

Further evidence of Temple's (2010, 689-690) misunderstanding on this point is given by the following:

> As in Simon and Levy (1963) and Simon (1979) they [Felipe and McCombie] examine the cross-section implications of the identity and show that it could lead to a (spurious) production relationship. The argument requires factor shares to be constant and the levels of factor prices to be similar across units. Felipe and McCombie relate this to international data [the Mankiw-Romer–Weil model, 1992] even though the assumption that factor prices are similar across countries is highly implausible. This does not strike Felipe and McCombie as a problem: "the critique does not rest on this assumption and so nothing depends upon whether or not it is correct. If the actual data do not have this property, then researchers who estimate the Cobb Douglas form… will not obtain a very good statistical fit". But their argument has veered off course. In these more general and plausible circumstances, a researcher no longer finds that Cobb-Douglas is a good fit. Instead the researcher concludes appropriately that a Cobb-Douglas technology does not provide a good explanation of the data in question. So what is the problem here? The proposed 'tyranny 'of the accounting identity seems part-time at best.

The quotation above shows that Temple misunderstands our arguments concerning the Mankiw *et al*., (1992) paper. Let us restate Mankiw *et al.*'s procedure. They posit a "world" aggregate production function that is a Cobb-Douglas. They see no problems in, say, aggregating Indian agriculture with the plough and oxen, the highly mechanized agricultural sector of the US and Europe, the aerospace industry of Europe and the US, the retailing sector with the hypermarkets in the developed countries and the bazaars of the less developed countries.

They, therefore, assume that the "world elasticity of substitution" is a meaningful concept and that all countries have access to the same level of technology. Commonsense (and a cursory acquaintance with aggregation theory) would suggest that this is not a sensible approach.

But nevertheless, Mankiw *et al.,* find that estimating their specification gives, in these circumstances, a reasonably good, but not perfect, statistical fit in terms of the R^2, and the coefficients are statistically significant. (The results using OECD data were poor.) As they use a (neoclassical) Cobb-Douglas production function and assume initially that growth is at its steady-state rate, they implicitly assume a constant capital-output ratio. Later in the paper they introduce a specification purporting to capture the non steady-state growth behavior of the countries.

The question is why do the data give such a reasonably good fit? Is it that the data has not refuted their assumptions underlying the concept of world production function? The answer is no. The reason is that Mankiw *at al.,* use value data, and cannot escape the fact that the series of value added, employment, and capital are related through the accounting identity. Looking at their data set, it is apparent that factor shares are empirically roughly constant. This is an empirical observation or one of Kaldor's stylized facts, not a "maintained hypothesis" as Temple asserts. In Solow's (2000, 2) words, "the ratio of capital to output shows no systematic trend".

What we show is that the initial less than perfect statistical fits of the Mankiw-Romer-Weil model is not because the capital-output ratio or factor shares show considerable variation, but because the wage rate varies considerably between countries. Mankiw *et al*., assume a constant level of technology, A_t, where, from the identity, we know that $A_t \equiv B w_t^{a} r_t^{(1-a)}$. We are fully aware that the "assumption that factor prices are similar across countries is highly implausible" as Temple (2010, 689) notes, implicitly criticizing us. Indeed, we discuss this at length in both our critique of Mankiw *et al*. (1992) (see Felipe and McCombie, 2005) and in Felipe and McCombie (2010a, 677). As the accounting identity holds separately for each country (both advanced and less developed nations), then we know immediately that the specification of the model of Mankiw *et al.,* with a constant level of technology, will not lead to a particularly good statistical fit. This indeed proves to be the case. As we point out (Felipe and McCombie 2010, 676), the identity shows that the assumption of both a constant technology and a spatially invariant rate of technical progress (i.e., *alnw + (1-a)lnr* and $a \ln \hat{w} + (1 - a)\hat{r}$ respectively) by Mankiw *at al.,* will produce a less than perfect statistical fit. If the capital-output ratio did show considerable variation, then the identity shows that Mankiw *et al.*'s specification is likely to give a poor fit to the data, not that we can suddenly be confident that we can find a specification where the data is actually estimating a "true" production function.

Mankiw *et al.*, (1992) improve the fit by including a human capital variable derived from school enrollment rates. As this is likely to be correlated with the wage rate, it acts as a proxy for the latter in the identity. Once the variation in factor prices is allowed for by regional dummies or is explicitly included in the regression, the Cobb-Douglas gives a good fit without, in the latter case, the need to include human capital, which Mankiw *et al.,* are forced to resort to (Felipe and McCombie, 2005a). Moreover, the estimated neoclassical speed of convergence becomes infinite. But our argument does not impose *a priori* the assumptions that factor shares are constant, or of a constant capital-output ratio. We know from the data, given these stylized facts hold, that the accounting identity tells us the Mankiw *et al.,* model is bound to give a good statistical fit to the data (subject to the variability of the real wage rate), before a single regression is run. Felipe and McCombie (2005a) confirm this by regression analysis. It is difficult to see any rationale for Temple's *non sequitur* that at this point our discussion "veers off course" (Temple 2010, 690). Empirically, if factor shares did vary considerably and we found another functional form that provides a better approximation to the identity than the Cobb-Douglas, it does not mean, as we have repeatedly emphasised, that we can now be confident that we are estimating an aggregate production function.

The concluding sentences of the above quotation of Temple (2010, p.690) demonstrate a fundamental confusion and it is worth repeating them:

> In these more general and plausible circumstances, a researcher no longer finds that Cobb-Douglas is a good fit. Instead the researcher concludes appropriately that a *Cobb-Douglas technology* does not

> provide a good explanation of the data in question. So what is the problem here? The proposed 'tyranny 'of the accounting identity seems part-time at least (emphasis added).

This implies Temple considers that if the data provide a good fit to the Cobb-Douglas, the researcher can conclude that a Cobb-Douglas *technology* does provide a good explanation. The "problem here" is that the whole point of the critique is that the existence of the accounting identity shows that no such inference can be made. The corollary is that if, for example, factor shares vary, we cannot suddenly be confident that an aggregate production function, *pace* Temple, is being estimated.[11]

Temple continues with this line of circular reasoning when he maintains that a constant capital-output ratio "makes little sense in the context of the Solow model. The Solow model can be seen precisely as a theory of adjustment to an equilibrium capital-output ratio. It makes little sense to reject estimates of the model on the basis of a highly restrictive assumption, even less so when that assumption rules out the central mechanism of the model" (Temple 2010, 690). This is again a case of the *petitio principii* fallacy because, as we have noted, the correct measure in Solow's growth model is the *physical* capital-output ratio. Of course, because of the heterogeneity of physical output and capital goods, there is no such thing as an aggregate physical capital-output ratio. See also Simon (1986, 172-183, Appendix A, "A Constant Long-Run *K/Y* Ratio is a Meaningless Observation") for a discussion of why the constant-price monetary value of the capital-output ratio will always tend to be approximately constant regardless of what is happening to the various individual physical capital-output ratios. If the data cannot show whether or

[11] Temple (2010, p.687) argues that "if the data have been generated by a translog, a simpler model such as the Cobb-Douglas will be an imperfect fit. This is because the output elasticities and factor shares will not be constant over time or across production units. Even if the researcher lacks information on factor shares, standard methods can be used to detect and investigate parameter heterogeneity that has been left unmodelled. The only way Felipe and McCombie can get around these arguments is to rule them out, by assuming that production relationships never exist, and factor shares behave in particular ways." The point to notice is that Temple again *assumes* that an aggregate production function exists in the form of a translog, which entirely begs the question under discussion. If the shares do show variability then, of course, the Cobb-Douglas relationship (not "production function") will give an imperfect fit and a more flexible functional form (not "production function") is needed.

not the aggregate production function exists, then the same applies to the whole Solow growth model, upon which it depends. It is not a case of rejecting the estimates of the parameters of the model – we know exactly what the estimates of the model are; they are the factor shares, but they cannot be interpreted as the physical aggregate output elasticities. Again, Temple misunderstands the argument.

To summarise: all Felipe and McCombie (2005a) does is to show, using the identity, the circumstances under which the augmented Solow model (the specification that Mankiw *et al.,* (1992) estimated) will lead to good results. We do not claim that the assumptions about the constancy of the wage and profit rates, the factor shares, the capital-output ratio, are correct theoretically or empirically (although some of them, as we have argued, are stylized facts in the literature). What we argue is that Mankiw *et al.*'s (1992) regression will work *if and only if* these conditions are met as the estimate a Cobb-Douglas relationship. In fact, what we implied in our discussion about the identity was that their poor initial results derived from the fact that all these stylized facts about the data were not met (especially the constancy of the level and growth of "TFP"). If these assumptions about the data are not correct, then the equation Mankiw *et al.,* (1992) estimated, will give a poor statistical fit. This is exactly what originally happened, and our point was that this can be seen without the need for estimating any regression to explain why.[12] The identity also tells us how to improve the goodness of fit.

[12] It is difficult to understand the implications of Temple's remark (2010, fn. 7, 691) noting the fact that the model of Mankiw *et al.,* can be expressed as a function of the logarithm of the aggregate labour share. He comments "But that quantity has been assumed constant across countries, so their [Felipe and McCombie's] suggested reinterpretation seems internally inconsistent (Temple 2010, fn 7, 691). This result was actually proved by Felipe and McCombie (2005, 375) and discussed there. Felipe and McCombie prove that if the stylized facts mentioned in their paper hold, then the model of Mankiw *et al.* reduces to the logarithm of the aggregate labour factor share. But note that this share does not have to be constant across countries. This theoretical result arises from the fact that the accounting identity is replicating the model of Mankiw *et al.,* which assumes *identical* output elasticities (and, hence, from the identity, identical factor shares). The same result arises for each country if we start from the case where the elasticities and, hence the shares, *differ* between the individual countries, although this will reduce the goodness of fit of the cross-country regressions. It is difficult to see how this is any way internally inconsistent. (The only interpretation we can make is that Temple is implicitly assuming that if factor shares differ for some unspecified reason, the regressions are estimating a "true" aggregate production function.)

There is no econometric solution to the implications of the critique.

As we are dealing with a misspecified representation of an accounting identity, the solution is *not* to use instrumental variables, or any other econometric technique (as used by, for example, Olley and Pakes, 1996, Blundell and Bond, 2000, and Levinsohn and Petrin, 2003. Temple also cites Ackerberg *et al.,* (2006) as an example of progress towards solving this identification problem, but again they assume the existence of an aggregate production function. The poor statistical results are not due to standard econometric problems (the identity does not have an error term). Poor statistical fits using time-series data are often found even though factor shares are relatively constant. The problem, as noted earlier, is that $a_t\hat{w}_t + (1 - a_t)\hat{r}_t$ is not constant, but has a pronounced cyclical component. Consequently, its approximation by the constant term in the regression in growth rates (or a linear time trend when log-levels are used) is responsible for the poor results. In these circumstances, it is necessary to find the correct approximation to this equation (e.g., through a different type of time-trend such as a trigonometric function).[13] If factor shares vary greatly then the Cobb-Douglas form will also give a poor statistical fit. In this case, a more flexible functional form for the identity is required.

The problem cannot be solved by disaggregating the value measures of capital (and output).

Disaggregation of labor and capital services as in Jorgenson and Griliches (1967) does not solve the problem. Temple argues that with sufficient disaggregation the aggregate production function may (or presumably may not) exist and all that remains is a statistical problem of correctly specifying its functional form. This is also incorrect. As we clearly stated in Felipe and McCombie (2010a, 673-674), disaggregating the relationship between output, labour, and capital as

$$V_i = f_i(L_{1i} , L_{2i}, \ldots L_{mi}; J_{1i}, J_{2i} \ldots J_{ni}) \quad (7)$$

[13] Adjusting the inputs, especially capital, for differences in capacity utilisation will also have the same effect.

does not solve the problem.[14] The production function can only be estimated if the disaggregation is such that physical quantities are used. The problem is not that the inputs are badly measured in value terms, but that they have to be measured in value terms in the first place.

The question is not so much about disaggregation, but the type of data, value versus physical. As we have argued, although not exempt from problems, with data in physical terms it is possible to estimate the technical parameters. Temple argues that "if the inputs have been disaggregated appropriately, then a production function may well exist, and the only remaining problem is a purely statistical one: can the data be used to establish the form of the relationship?" (Temple 2010, 687). Temple argues that, provided we sufficiently disaggregate the constant-price value data of the capital stock and employment, the resulting aggregate production function exists and therefore can be estimated using value data.[15] He does have the proviso that the correct measure of total factor productivity is required, which he sees as a difficult, but not insuperable problem. This legerdemain occurs in his 2006 paper and he repeats it in his 2010 comment. Thus, he seems to consider that the critique rests on a "fundamental identification problem" (Temple 2010, 685). However, we have long argued that, *pace* Temple, it is not a statistical identification problem if this implies that it is possible, in principle, to specify a model where the aggregate production function can be statistically refuted. Temple (2010, 687), paradoxically also recognizes that "the argument is not simply one about statistical identification."[16]

To reiterate our 2006 argument: the problem is that, to be meaningful, production functions must be estimated using physical quantities. As these are heterogeneous and have to be summed, constant price value data have to be used. Consequently, no matter how many inputs (and outputs) are specified and measured in value terms, the problem posed

[14] Temple's argument is puzzling, as he accepts that our arguments are not about input mis-measurement, but about the dangers of using value added to measure output, and constant price value data to measure the capital stock.

[15] Temple only concentrates on the disaggregation of inputs, although his argument must logically apply to the different outputs, which means that we have to disaggregate the aggregate production function.

[16] However, from a careful reading of the text, it is not clear if he merely correctly attributes this to us, or whether he accepts that it logically follows from our critique.

by the identity still arises. Jorgenson and Griliches (1967, 253) (see Felipe and McCombie, 2006) start out by *assuming* the existence of an aggregate production function, perfect competition, and that factors are paid their marginal products. They use this approach to disaggregate the constant price value indices of the capital stock in order to try empirically to eliminate the residual. Jorgensen and Griliches (1967, footnote 2) explicitly state that because of their assumptions, their approach cannot be used to *test* the marginal productivity theory of factor pricing.

Simulation results confirm the importance of the critique.

As part of our 2010a reply to Temple we cite a simulation study of ours where we show that with a constant mark-up pricing the data will give a perfect fit to a Cobb-Douglas production function, where the estimated coefficients of the log of capital and labor are 0.75 and 0.25, respectively; while the true output elasticity were 0.25 and 0.75, respectively. We assume the existence of well-defined physical micro-production functions not because we necessarily believe they exist, but to show the implications of the critique even under these circumstances.

Temple argues that as the estimated coefficients of the log of capital and labor using value data differ markedly from the true output elasticities, there must be large differences between the rewards to factors and their marginal products. "Those are not the usual assumptions made in interpreting the results from estimated production functions" (Temple 2010, 690). But any researcher with only access to the value data and interpreting the results of the estimated "production function" would find that the estimated "output elasticities" equal the factor shares. Thus, the neoclassical researcher would erroneously conclude that markets are perfectively competitive, constant returns to scale prevail, and that factors are paid their marginal products. This would also be the case, as we show in our simulations, when the true production function displays increasing returns, or, indeed, there is no well-defined relationship between the outputs and inputs. In the simulation model we use (Felipe and McCombie, 2006), prices are determined by a mark-up on unit costs, which in turn is determined by, for example, the state of competition in the industry and the relative power of labor and capital in the wage bargaining process. It may well differ from the physical marginal productivity of labor if the firm (but not the researcher) knows the true micro production function, but so what?[17] Firms, under neoclassical assumptions, will set the rewards equal to the marginal product measured in value terms and are unlikely to know a worker's physical marginal product. (Moreover, there are vast sectors of the economy where there is no reliable independent measure of output even in constant-price value terms.)

There have been a number of other important simulation studies which demonstrate how the data will give a good fit to a Cobb-Douglas, even though we know by the construction of the hypothetical data this is not reflected in the underlying technology. These include a study where the micro-production functions deliberately violate the conditions for successful aggregation (Fisher, 1971); where the production function has a fixed-coefficients technology (Shaikh, 2005); and where firms satisfice, rather than optimize (Nelson and Winter, 1982). For a discussion of these studies, see Felipe and McCombie (2010b).

6. Conclusions

The Cambridge capital theory controversies and the related aggregation problems have had no bearing on the use of the aggregate production function, which continues to be widely and uncritically used. We suggest that the answer to this conundrum is the instrumental justification that in practice it works. However, the fact that very simple functional forms and two highly aggregate variables (with the constant price value of the capital stock in particular subject to all kinds of statistical measurement errors) can often explain over 90 percent of the variation in output is due simply to the fact that the three variables

[17] Temple argues that some argue that "no firm knows its production function" but he considers it knows its costs and that well-behaved cost functions are mirrored by the existence of production functions. But a cost function is also derived from the accounting identity and will be mirrored by a "spurious" production function. (See Felipe and McCombie 2011-12). A neoclassical cost function does not guarantee the existence of a well-behaved production function.

are definitionally related. This explanation does not depend upon any specific assumptions such as constant factor shares, a constant weighted log-level (or growth) of the average of the wage and profit rates, or a constant capital-output ratio. Allowing these to vary does not mean that all the aggregation problems and the problems posed by the accounting identity disappear, and that we can be confident of estimating a technological relationship.

The key disagreement between Temple and us is that we argue using value data that a researcher can always find a perfect fit to the data, with the estimated coefficients equal to the factor shares (and not only when these are constant), even though no aggregate production function exists. Temple does not share this conclusion. We have shown that the only reason why factor shares and the output elasticities may differ is that the specific functional form estimated does not accurately track the accounting identity. Temple unwittingly concedes our case when he states: "Moreover, the production function *may* appear simple and well-behaved even when no *stable relationship* exists and the true extent of the misspecification may never be detected" (Temple 2010, 689; italics added). This accurately summarises our position, although we argue it is not a case of 'may', but of 'will'; and furthermore, that the statement 'no stable relationship' includes the case when plausibly the aggregate production function does not exist. An implication of the above quotation is that the researcher can never know whether or not the estimates of the aggregate production function mean anything. As we noted above, Temple correctly states, "the argument is not simply one of statistical identification" (Temple 2010, 687), but then inconsistently and erroneously states that "to the extent that a researcher can control for the variation in TFP and takes care over the specification, the simultaneous existence of the value added identity does not invalidate these methods". Unfortunately for the researcher it does and Temple has not demonstrated otherwise.

Anwar Shaikh's (1974) important conclusions about the accounting identity critique have stood the test of time.

References

Acemoglu D. (2009), *Introduction to modern economic growth*, Princeton University Press, Princeton.

Ackerberg D. A., Caves J. K., Frazer G. (2006), *Structural identification of production functions*, retrieved at http://folk.uio.no/rnymoen/Ackerberg_Caves_Frazer.pdf on June 12, 2013.

Aghion P., Howitt P. (1998), *Endogenous growth theory*, MIT Press, Cambridge (MA).

Aghion P., Howitt P. (2009), *The economics of growth*, MIT Press, Cambridge (MA).

Arrow K. J., Bernheim B. D., Feldstein M. S., McFadden D. L. , Poterba J. M., Solow R. M. (2011), *100 years of the American Economic Review: the top 20 articles*, American Economic Review, 101(1), 1-8.

Barro R. J., Sala-i-Martin X. (1995), *Economic growth*, 2nd edition 2004, McGraw-Hill, New York.

Basu S., Fernald J. G., Kimball M. S. (2006), *Are technology improvements contractionary?*, American Economic Review, 96(5), 1418-1448.

Birner J. (2002), *The Cambridge controversies in capital theory: a study in the logic of theory development*, Routledge, London.

Blundell R., Bond S. (2000), *GMM estimation with persistent panel data: an application to production functions*, Econometric Reviews, 19(3), 321-340.

Champernowne D.G. (1953-1954), *The production function and the theory of capital: a comment*, The Review of Economic Studies, 21(2), 112-135.

Cobb C. W., Douglas P. H. (1928), A theory of production, American Economic Review, 18(1), 139-165.

Cohen A., Harcourt G. C. (2003), *Retrospectives: whatever happened to the Cambridge capital theory controversies?*, Journal of Economic Perspectives, 17(1), 199-214.

Douglas P.H. (1967), *Comments on the Cobb-Douglas production function*, in Brown M. (ed.), *Theory and empirical analysis of production: studies in income and wealth*, 31, National Bureau of Economic Research.

Douglas P. H. (1976), *The Cobb-Douglas production function once again: its history, its testing, and some new empirical values*, Journal of Political Economy, 84(5), 903-915.

Felipe J., Hasan R., McCombie J. S. L. (2008), *Correcting for biases when estimating production functions: an Illusion of the laws of algebra?*, Cambridge Journal of Economics, 32(3), 441-459.

Felipe J., McCombie J. S. L. (2002), *A problem with some recent estimations and interpretations of the markup in manufacturing industry*, International Review of Applied Economics, 16(2), 187-215.

Felipe J., McCombie J. S. L. (2005a). *Why are some countries richer than others? A sceptical view of Mankiw-Romer-Weil's test of the neoclassical growth model*, Metroeconomica, 56(3), 360-392.

Felipe J., McCombie J. S. L. (2005b), *How sound are the foundations of the aggregate production function*, Eastern Economic Journal, 31(3), 467-488, revised in Harcourt G. C. and Kriesler P. (eds.) (2013), *The Oxford Handbook of Post-Keynesian Economics*, Vol. 2, Critiques and Methodology, Oxford University Press, New York, 202-230.

Felipe J., McCombie J. S. L. (2006b), *The tyranny of the identity: growth accounting revisited*, International Review of Applied Economics, 20(3), 283-299.

Felipe J., McCombie J. S. L. (2007), *On the rental price of capital and the profit rate: the perils and pitfalls of total factor productivity growth*, Review of Political Economy, 19(3), 317-345.

Felipe J., McCombie J. S. L. (2009), *Are estimates of labour demand functions mere statistical artefacts?*, International Review of Applied Economics, 23(2), 147-168.

Felipe J., McCombie J. S. L. (2010a), *What is wrong with aggregate production functions. On Temple's aggregate production functions and growth economics*, International Review of Applied Economics, 24(6), 665-684.

Felipe J., McCombie J. S. L. (2010b), *On accounting identities, simulation experiments and aggregate production functions: a cautionary tale for (neoclassical) growth theorists*, in Setterfield M. (ed.), Handbook of Alternative Theories of Economic Growth, 189-207, Edward Elgar, Cheltenham.

Felipe J., McCombie J. S. L. (2011-12), *On Herbert Simon's criticisms of the Cobb-Douglas and CES production functions*, Journal of Post Keynesian Economics, 34(2), 275-294.

Felipe J., McCombie J. S. L. (2012), *The aggregate production function. "Not even wrong"*, Cambridge Centre for Economic and Public Policy, Department of Land Economy, University of Cambridge, WP 03-12.

Felipe J., McCombie J. S. L. (2013), *The aggregate production function and technical change: "not even wrong"*, Edward Elgar, Cheltenham.

Fernald J. G., Neiman B. (2010), *Growth accounting with misallocation: or doing less with more in Singapore*, NBER working paper no. 16043.

Ferguson C. E. (1969), *The Neoclassical theory of production and distribution*, Cambridge University Press (2nd edition, 1971), Cambridge (UK).

Fisher F. M. (1971), *Aggregate production functions and the explanation of wages: a simulation experiment*, The Review of Economics and Statistics, 53(4), 305-325.

Fisher F. M. (1992), *Aggregation. Aggregate production functions and related topics*, Monz J. (ed.), Harvester Wheatsheaf, Business & Economics, London.

Fisher F. M. (2005), *Aggregate production functions: a pervasive, but unpersuasive fairytale*, Eastern Economic Journal, 31(3), 489-491.

Friedman M. (1953), *The methodology of positive economics*, in Friedman M. (ed.), *Essays in positive economics*, Chicago University Press, Chicago.

Gandolfo G. (2008), *Comment on 'C.E.S. production functions in the light of the Cambridge critique'*, Journal of Macroeconomics, 30(2), 798-800.

Goodwin R. M. (1967), *A growth cycle*, in Feinstein C. H. (ed.), *Socialism, capitalism and economic growth*, Essays presented to Maurice Dobb, Cambridge University Press, Cambridge (UK).

Haache G. (1979), *The theory of economic growth: an introduction*, Macmillan, London.

Hall R. E. (1988), *The relation between price and marginal cost in U.S. industry*, Journal of Political Economy, 96(5), 921-947.

Han Z., Schefold B. (2006), *An empirical investigation of paradoxes: reswitching and reverse capital deepening in capital theory*, Cambridge Journal of Economics, 30(2), 737-765.

Harcourt G. C. (1972), *Some Cambridge controversies in the theory of capital*, Cambridge University Press, Cambridge (UK).

Hoover K. D. (2012), *Applied intermediate macroeconomics*, Cambridge University Press, Cambridge (UK).

Jones C. S. (1998), *Economic growth*, WW Norton (second edition 2002), New York.

Jones H. G. (1975), *An introduction to modern theories of economic growth*, Nelson, Middlesex.

Jorgenson D. W., Griliches Z. (1967), *The explanation of productivity change*, Review of Economic Studies, 34(99), 249-283.

Kincaid H. (1996), *Philosophical foundations of the social sciences: analyzing controversies in social research*, Cambridge University Press, Cambridge (UK).

King J. E. (2002), *A history of post Keynesian economics since 1936*, Edward Elgar, Cheltenham.

King J.E. (ed.) (2012), *The Elgar companion to post Keynesian economics*, Edward Elgar (second edition), Cheltenham.

Kuhn. T.S. (1970), *The structure of scientific revolutions*, University of Chicago Press (second edition), Chicago.

Levhari D. (1965), *A nonsubstitution theorem and switching of techniques*, The Quarterly Journal of Economics, 79(1), 98-105.

Levinsohn J., Petrin A. (2003), *Estimating production functions using inputs to control for unobservables*, The Review of Economic Studies, 70(2), 317-341.

Mankiw N. G., Romer D., Weil D. N. (1992), *A contribution to the empirics of economic growth*, The Quarterly Journal of Economics, 107(2), 407-437.

Nelson R. R., Winter S. G. (1982), *An evolutionary theory of economic change*, Harvard University Press, Cambridge (MA).

Olley S., Pakes A. (1996), *The dynamics of productivity in the telecommunications equipment industry*, Econometrica, 64(6), 1263-1298.

Phelps Brown E.H. (1957), *The meaning of the fitted Cobb-Douglas function*, The Quarterly Journal of Economics, 71(4), 546-60.

Robinson J.V. (1953-54), *The production function and the theory of capital*, The Review of Economic Studies, 21(2), 81-106.

Robinson J. V. (1956), *The accumulation of capital*, Macmillan, London.

Robinson J. V. (1970), *Capital theory up to date*, Canadian Journal of Economics, 3(2), 309-317.

Samuelson P. A. (1962), *Parable and realism in capital theory: the surrogate production function*, The Review of Economic Studies, 29(3), 193-206.

Samuelson P. A. (1963), *Problems of methodology - Discussion*, American Economic Review, Papers and Proceedings, 53(2), 231-236.

Samuelson P. A. (1966), *Summing up*, The Quarterly Journal of Economics, 80(4), 568-583.

Shaikh A. (1974), *Laws of production and laws of algebra: the Humbug production function*, The Review of Economics and Statistics, 56(1), 115-20.

Shaikh A. (1980), *Laws of production and laws of algebra: Humbug II*, in Nell E. J. (ed.), *Growth, profits and property*, Cambridge University Press, Cambridge (UK).

Shaikh A. (2005), *Nonlinear dynamics and pseudo-production functions*, Eastern Economic Journal, 31(3), 447-466.

Simon H. A. (1979), *On parsimonious explanations of production relations*, The Scandinavian Journal of Economics, 8(4), 459-474.

Simon H. A., Levy F. K. (1963), *A note on the Cobb-Douglas function*, The Review of Economic Studies, 30(2), 93-94.

Simon J. L. (1986), *Theory of population and economic growth*, Basil Blackwell, Oxford (UK).

Solow R. M. (1957), *Technical change and the aggregate production function*, The Review of Economics and Statistics, 39(3), 312-320.

Solow R. M. (1974), *Laws of production and laws of algebra: the Humbug production function: a comment*, The Review of Economics and Statistics, 56(1), 121.

Solow R. M. (1988), *Growth theory and after*, American Economic Review, 78(3), 307-317.

Solow R. M. (2000), *Growth theory: an exposition*, Oxford University Press (second edition), Oxford (UK)

Sraffa P. (1960), *Production of commodities by means of commodities: prelude to a critique of economic theory*, Cambridge University Press, Cambridge (UK).

Temple J. R. W. (2005), *Dual economy models: a primer for growth economists*, Manchester School, 73(4), 435-478.

Temple J. R. W. (2006), *Aggregate production functions and growth economics*, International Review of Applied Economics, 20(3), 301-317.

Temple J. R. W. (2010), *Aggregate production functions, growth economics, and the part-time tyranny of the identity: a reply to Felipe and McCombie*, International Review of Applied Economics, 24(6), 685-692.

Temple J. R.W., Wößmann L. (2006), *Dualism and cross-country growth regressions*, Journal of Economic Growth, 11(3), 187-228.

Wan H. Y. (1971), *Economic growth*, Harcourt Brace Jovanovich, New York.

Weil D. N. (2005), *Economic growth*, Pearson, Addison Wesley, Boston.

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HUMBUGS AND OTHER EXOTICA

Celebrating Anwar Shaikh

"However, we cannot assess the adequacy of theories by counting footnotes♥. What I should like to do here is to consider the logical and theoretical status of the production function evidence."
(Simon, 1979, p. 461)

Abstract

In this article the author tries to 'pick and choose' a few choice items from the vast canvas on which Anwar Shaikh has sketched his philosophy and methodology of analytic economic theory. The author further comments on these items, 'here and there', and scrutinizes them critically from one or another point of view. Thus, the author offers a brief 'excursus' on the HUMBUG and its distinguished predecessors. Finally, the author indulges in some elementary and exotic adventures in the nonlinear dynamics used in Shaikh (2005). A similar exercise is attempted on the basis of the author's reflections on Shaikh's provocative and important essay on 'Rethinking Microeconomics' (Shaikh, 2012) and his recent allegiance to reflexivity (Shaikh, 2010) - in the senses given to it by Soros (2008) - and emergence.

JEL Classification: A11, B21, B31, B51, C02, D50.

Keywords: Humbug Economy, Rethinking Microeconomics, Lotka-Volterra System, Reflexivity, Disequilibrium Dynamics.

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♥ Simon is referring to the 345 (sic!) references in Walters (1963).

1. A prologue

"[D]ata is never just a collection of pre-existing facts. Theory always intervenes, not merely in the interpretation of events, but in their very construction (and occasionally in their suppression, as we know only too well) […] Data is never innocent."
(Shaikh 2013, pp. 4-5; italics added).

In a personal letter to Sraffa, soon after reading Production of Commodities by Means of Commodities (Sraffa, 1960), John Hicks wrote:

"Economic theory (teachable economic theory, at least) was getting just a bit boring lately: for the second time in your life you have livend it up again. Thank you." Hicks (1960).

Sraffa 'livened' up 'economic theory' first in 1926 (Sraffa, 1926) and, then, for the second time, thirty four years later, in 1960. Shaikh did so, first in 1974 (although the first version saw official light of day two years before that[1]) and, now, almost forty years later with his *magnum opus* (Shaikh, 2013), which is, surely, to become a classic critique of every kind of orthodox complacency in economic theory. Many monographs and articles reflected on 'Marx after Sraffa', 'Keynes after Sraffa' – extending to copycat inanities like 'Keynes after Lucas', and so on. There should have been, with more reason, pungent essays on 'Surrogates after Humbugs' – but there were none; I hope there will, however, be serious reflections on Political Economy after Shaikh (2013), for it is a serious and immanent critique of the political and value theoretic foundations of orthodox economic theory, capable of 'livening up' a complacent, somnambulant, economic theory.

Over the many years in which I have felt educated and enlightened by reading Anwar Shaikh's systematic writings on foundational issues, mostly of macroeconomics, but recently also of microeconomics (Shaikh, 2012), it has seemed to me to be possible to summarise (the

[1] The poignant, even melancholy, saga of the vicissitudes of that classic by a youthful Shaikh is sympathetically narrated in Harcourt's wonderful contribution to this *Special Issue* of Global & Local Economic Review in honour of Anwar Shaikh, also commemorating the award of the *Fondazione Pescarabruzzo NordSouth Award in the Social Sciences*, for 2013, to him.

unsummarisable) his guiding, disciplining, analytical concerns under the following eight categories[2].

i. The Determination of Prices & Profits
ii. The Impact of Technical Change on Profitability
iii. The Political Economy of National Income Accounts
iv. The Impact of State Taxation and Expenditures on Labour Income
v. On the Macrodynamics of Effective Demand in a Growth Context
vi. On a Classical Explanation of Inflation
vii. On a Classical Explanation International Trade and Exchange Rates
viii. On the Determination of Stock Prices and Interest Rates by means of the Equalization of Profit Rates across Sectors

The central concerns of Shaikh's theoretically based empirical work has been the attempt to understand the fundamental processes at work in advanced capitalism, categorized in the answers he seeks for the following questions:

1. *How do market economies work, and why do they generate certain patterns which seem to cut across differences in origin, in culture, and even historical epochs?*
2. *Why is capitalist growth characterized by order-within-disorder, periodically punctuated by episodes of general economic crisis?*
3. *Why is unrestrained capitalist development so typically uneven across nations, across regions, and across individuals?*

In approaching such questions, he claims to have always found it crucial that one start from a solid theoretical foundation grounded in the actual phenomenon of the object of one's investigation.

This vision has now seen a welcome appearance in the form of the monumental new monograph (Shaikh, 2013), for which many of us have been waiting a long time.

[2] I rely on Shaikh (2000) for this and the immediately following characterisations of his visions, work and framework for analytical economic theory.

I do not aim, in this very modest essay, to survey the vast canvas on which Shaikh has sketched his philosophy and methodology of analytic economic theory. I try only to 'pick & choose' a few choice items from that panorama of vistas, comment on them, 'here and there', and scrutinize them critically from one or another point of view – one that will not be strange or uncomfortable for him.

Thus, the next section is a brief 'excursus' on the HUMBUG and its distinguished predecessors. In section 3 I indulge in some elementary and exotic adventures in the nonlinear dynamics used in Shaikh (2005). A similar exercise is attempted on the basis of my reflections on Shaikh's provocative and important essay on 'Rethinking Microeconomics' (Shaikh, 2012) and his recent allegiance to *reflexivity* (Shaikh, 2010) – in the senses given to it by Soros (2008) – and *emergence*.

I am, of course, well aware that this is an exercise in *scratching* a deep and complex *surface*, from which I am attempting to infer and interpret visions and vistas of a seriously committed scholar.

2. The HUMBUG and (some of) its precursors

"In the critic's vocabulary, the word "precursor" is indispensable, but it should be cleansed of all connotations of polemic or rivalry. The fact is that every writer creates his own precursors. His work modifies our conception of the past, as it will modify the future."
(Borges J. L. [1951] (1985) p. 108).

It is not often explicitly recognized that the unfortunate – and, as Shaikh has amply demonstrated, untenable - 'Cobb-Douglas' production function formulation originated in one of Wicksell's lesser known (most likely because it was in Swedish) contributions to economic analysis[3]. Moreover, it is – in spite of sterling efforts by Shaikh, Simon, Phelps Brown and a host of others – more often forgotten that

3 Wicksell (1900), pp. 305 – 337, although it was already mentioned in a letter to his younger colleague, Gustaf Steffen one year earlier (cf. Gårdlund, 1958, p. 331). To Wicksell's credit, it must be pointed out that he never used 'capital' as one of the factors of production, in his specification of a production function with what is now known as the Cobb-Douglas formulation. His factors of production were, true to his Austrian capital theoretic 'heritage', land, labour and time.

Paul Douglas actually tried to find a functional formulation to 'explain' *the constancy of the wage share in total national income* (output).

That it was transmogrified into a production function of a particular linear homogenous specification – eventually turned into the infamous and ubiquitous Cobb-Douglas formulation

– was entirely due to a conjunction of an apologetic theory of distribution – the neoclassical marginal productivity theory of functional income distribution - and a particular interpretation of *Euler's theorem on homogeneous functions*.

Thus it was that Anwar Shaikh returned the issue to its 'womb', so to speak: *"What is not obvious is, however, is that so long as aggregate shares are constant, an aggregate Cobb-Douglas function having apparently 'constant returns to scale' will always provide an exact fit, for any data whatsoever."* (Shaikh 1974, p. 116, italics added).

Wicksell (1900), pp. 305 – 337, although it was already mentioned in a letter to his younger colleague, Gustaf Steffen one year earlier (cf. Gårdlund, 1958, p. 331). To Wicksell's credit, it must be pointed out that he never used 'capital' as one of the factors of production, in his specification of a production function with what is now known as the Cobb-Douglas formulation. His factors of production were, true to his Austrian capital theoretic 'heritage', land, labour and time.

His dramatic use of *The Humbug Economy* (op.cit, p. 118; figure 1) to demonstrate this 'truism' seems not to have deterred legions of neoclassical economists from ascribing almost mystical powers to the production function. As Tjaling Koopmans – no unorthodox non-neoclassical by any standards – pointed out, almost pungently: *"With princely unconcern econometricians have continued to fit aggregate production functions approximating an aggregate output index, for an economy or a sector, by a function F(K,L) of aggregate labour (L) and capital (K) input indices. When the matter of the logical foundations for such a construct is raised, words such as 'parable'4 or 'metaphor' are pressed into service."*(Koopans 1977, p. 144, italics added).

Shaikh (and Simon[4]) questioned precisely 'the logical foundations for [the Cobb-Douglas] construct.

Simon, in his joint paper with Levy (Simon & Levy, 1963), a decade before Shaikh, in his typically acute way, had come to almost the same conclusion[5]. But it was Phelps Brown who is the Kafkian precursor *à la* Borges, to Shaikh's *HUMBUG*: *"The Cobb-Douglas k, and the share of earnings in income, will be only two sides of the same penny."*
(Phelps Brown 1957, p. 557, italics added).

Legions of non-orthodox economists have replicated, strengthened and expanded Shaikh's foundational result, but – to use Koopman's felicitous phrase – 'with princely unconcern' neoclassical economists continue to litter textbooks, even formally advanced versions of them (eg., Romer, 2006), with production functions of the Cobb-Douglas form, ignoring comprehensively the facts of other side of 'the same penny'. The 'dishonour' role of neoclassical economists who continue their 'princely unconcerns' include many who have gone on, and continue to go on, displaying utter disregard for foundational infelicities, when they come into conflict with ideologically grounded opinions, masquerading as 'scientific beliefs': all the way from Böhm-Bawerk and J.B. Clarke, to lesser contemporaries such as Levhari (1965), Romer (*op.cit*) and Brems (1977) – and the whole newclassical school, all of whom, without exception, feel quite comfortable with the dissonance between theoretical rigour (however defined) and empirical exercises.

That econometricians, with even less scrupulous standards, and much less allegiance to economic theoretic foundational consistency,

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[4] See the lead quote on the title page of this paper.

[5] As he pointed out with characteristic candour, in his letter to Marc Lavoie (Simon, 1985), fully over two decades after his own paper with Levy (ibid) and more than a decade after Shaikh's classic (italics added): "Professor Shaikh's derivation is very similar to mine for the time series case. … He starts with constant factor shares; I start with a constant saving rate and derive constant factor shares (assuming also a constant rate of interest). Hence our papers seem to me complementary in showing that the spurious fit of the Cobb-Douglas will occur under even broader ranges of assumptions that each of us originally thought."

indulge in fitting alleged aggregate production data to derive meaningful (sic!) exponents for the Cobb-Douglas specification should no longer be surprising. But that the practice has even spread to assuming Cobb-Douglas utility functions is nothing short of a theoretical scandal.

Shaikh's recent reflection (Shaikh, 2005, p. 447; italics added) on this dissonance perfectly summarises the issue: *"It is curious that a tradition so insistence on the necessity of micro-foundations should rely so heavily on a construction that cannot be derived from microfoundations. Defenders [like, for example, Brems (op.cit)] claim that aggregate production functions are worth retaining because they possess important virtues, and because they appear to work at an empirical level."* (Shaikh, 2005, p. 447; italics added).

The other side of the *utility penny*, by those who extend magical beliefs to utility analysis of choice, has *never*, to the best of my knowledge, been a subject of discourse even in non- orthodox circles.

3. 'Nonlinear dynamics and pseudo-production functions'[6]

"It has long seemed to me that Volterra's problem of the symbiosis of two populations – partly complementary, partly hostile – is helpful in the understanding of the dynamical contradictions of capitalism, especially when stated in a more or less Marxian form." (Goodwin, 1967, p. 55).

In an interesting exercise, Shaikh (2005, p. 448) compares two sets of data, one whose 'generating process is transparent and strictly non-neoclassical' and another 'whose generating process is the object of dispute.'

There are, however, two problems with the 'non-neoclassical generating process'. Firstly, it is not clear that it is unambiguously non-neoclassical, despite the claims of its original 'architect' – Richard Goodwin (as claimed in the lead quote of this section). Secondly, the 'transparency of the generating process' is not quite clear, for a very special technical reason.

[6] This is the title of Shaikh (2005).

The first issue is not of much concern for me, here, except in so far as to point out that Goodwin's labour productivity assumption can 'easily' be replaced by a Cobb-Douglas production function so that the resulting dynamics in the share of wages and the (un)employment ratio displays, qualitatively, the same dynamics as the original model.

As for the second issue, it is rarely – if ever – recognised by the many non-neoclassicals who use Goodwin (1967) as a fulcrum around which to construct, and empirically investigate, a variety of questions of aggregate dynamics that the *discretisation* of the *coninuous* nonlinear system of Lotka-Volterra equations requires delicate handling. Indeed, I know of NO application, in economics, of these equations, for empirical investigation of any kind of aggregate dynamics to be theoretically rigorous.

Moreover, the original dynamics of the Lotka-Volterra system is structurally unstable and, therefore, using it to generate data sets is fraught with too many sensitivities that are not easy to circumvent.

Consider the original Lotka-Volterra system of non-linear differential equations[7]

$$\frac{dx}{dt} = ax - axy \dots\dots\dots\dots\dots..(1)$$

$$\frac{dy}{dt} = -cy + cxy \dots\dots\dots\dots\dots..(2)$$

$$a > 0, c > 0$$

The 'usual' discretization of (1) & (2), for generating and studying empirical data, is:

$$\Delta x_n = a x_n (1 - y_n) \Delta t_n \quad \dots\dots\dots\dots..(3)$$

$$\Delta y_n = c y_n (x_n - 1) \Delta t_n \quad \dots\dots\dots\dots..(4)$$

[7] They should really be described as 'quasi-linear'. I shall rely heavily on the results and notations in Potts (1982), to which I was directed by Paul Samuelson, via his contribution to the Goodwin Festschrift, which I edited.

It seems to me that Shaikh has used the above [(3) & (4)] system of discretisations of (1) & (2) for the data generation process. Now, quite apart from the difficulties of using a structurally unstable system in *any* empirical analysis, it so happens that the above system of discretized equations, when used as the basis for analyzing the centre-type solutions of (1) & (2), generate, for $a \neq c$, 'spurious solutions with negative x_n or y_n.

An intuitive way out of this difficulty would be to replace (1) & (2) by:

$$\frac{d}{dt}(x - lnx) = -a(x-1)(y-1) \quad \ldots\ldots\ldots\ldots..(5)$$

$$\frac{d}{dt}(y - lny) = -c(x-1)(y-1) \quad \ldots\ldots\ldots\ldots..(6)$$

In the case of (5) & (6), the 'usual' discretisations can be used without any difficulty of spurious solutions and the like arising. However, the drawback here is clear: one loses the intuitive justification of an 'understanding of the dynamical contradictions of capitalism, especially when stated in a more or less Marxian form,' when (1) & (2) are replaced by (5) & (6).

Whether the non-neoclassical, 'more or less Marxian form' of the underlying economic model can be preserved, when trying to derive (5) & (6) as its – i.e., the economic model's - 'final equations', remains a moot question.

4. Rethinking microeconomics, reflexivity and disequilibrium dynamics

"If it [the foundation stones of microeconomics as it is taught today] is wrong, why not throw it away? Yes, I am throwing it away. I think the textbooks are a scandal. I think to expose young impressionable minds to this scholastic exercise as though it said something about the real world, is a scandal. … I find that inexcusable." (Simon, 1997, p. 397).

My former colleague, at Queen's University of Belfast, R.D.C Collison Black, distinguished scholar of impeccable intellectual integrity, once 'lamented' to me that he was 'tired of being referred to as the *Jevons man*.' It would be entirely understandable if Anwar

Shaikh, as distinguished a scholar of equally impeccable intellectual integrity, felt slightly weary of being referred to as '*the HUMBUG man*'!

In recent years, now extending to over a decade and a half, Shaikh has broached new frontiers of analytical economic investigations, and contributed to them in enlightening ways, whilst remaining faithful to the vistas and visions I outlined in the opening section.

I identify three disciplining criteria informing Shaikh's current frontiers of research – of course there may well be more; or, indeed, my characterisations completely incorrect – in furthering the foundations of economic analysis, with solid grounding in empirical facts, sometimes euphemistically described as 'reality'. In all these, Shaikh, like Simon, shuns any and every kind of 'Armchair Economics'.

First of all, there is his lifelong concern with 'the fallacy of composition', which I have come to call the *Mereological Confusion*[8].

Secondly, there is the increasing concern and emphasis in his writings for one or another kind of notion of *emergence* to be at the analytical core of the natural process analysis of his economics, whether it is macroeconomics or microeconomics.

Thirdly, he has come to advocate a particular notion of reflexivity, especially in the analysis of the financial-real nexus of economic dynamics, with some kind of microeconomic underpinning. The notion of reflexivity Shaikh seems to subscribe to is the one introduced by Soros: *"As a way of explaining financial markets, I propose an alternative paradigm that differs from the current one in two respects. First, financial markets do not reflect prevailing conditions accurately; they provide a picture that is always biased or distorted in one way or another. Second, the distorted views held by market participants and expressed in market prices can, under certain circumstances, affect the so-called fundamentals that market prices are supposed to reflect. This two-way circular connection10 between market prices and the underlying reality I call reflexivitty."* (Soros, 2008, §2; italics added).

8 Mereology is the logic of part/whole relations.

I subscribe to a different interpretation of both *emergence* and *reflexivity* and, therefore, also of the *mereological confusion* or fallacy (the *fallacy of composition*). My interpretation is a unified conception of all three, based on their foundations in Herbert Simon's kind of cognitive science.

Reflexivity, for example, in this interpretation, is not about 'explaining' facts, whether observed in financial market behaviour or constructed theoretically. This alternative view is elegantly described by Cohen-Cole (2005): *"[T]heory construction and creative problem-solving was the cognitive scientists' model of everyday thinking and problem-solving. Learning was not so much a process of acquiring facts about the world as of developing a skill or acquiring proficiency with a conceptual tool that could then be deployed creatively […] . For instance, according to […] Noam Chomsky, […] a child learning a language was not acquiring specific words so much as operating like a scientist by actively developing a theory of how to speak properly."* (Cohen-Cole, 2005, p. 122; italics added).

It is this aspect of reflexivity that is developed by Herbert Simon in his lifelong research program in offering an alternative to the Armchair Theorising of orthodox economists.

Similarly, *emergence*, with its roots in the work of the *British Emergentists* and their underpinning of it in considerations of the *evolution of mind*, has a natural cognitive science basis.

As for the mereological confusion, in its cognitive science interpretation, it is best described in the context of neuroscience, especially since the reactionary, uncompromisingly reductionist nature of neuroeconomics: *"Mereology is the logic of part/whole relations. The neuroscientists' mistake of ascribing to the constituent parts of an animal attributes that logically apply only to the whole animal we shall call 'the mereological fallacy' in neuroscience. The principle that psychological predicates which apply only to human beings (or other animals) as wholes cannot intelligibly applied to their parts, such as the brain, we shall call 'the mereological principle ' in neuroscience. Human beings, but not their brains, can be said to be thoughtful or to be thoughtless animals; animals, but not their* *brains, let alone the hemispheres of their brains, can be said to see, hear, smell and taste things; people, but not their brains, can be said to make decisions or to be indecisive."* (Bennett & Hacker, 2003, p.22, italics added)

If I may summarise this particular Simonian vision of the interaction and interdependence, from a cognitive science viewpoint, of the triptych of the mereological fallacy, emergence and reflexivity, than I may say, with confidence, it is about the evolutionary, disequilbrium dynamics, of economic processes, at any level.

The problem to be overcome by anyone subscribing to this cognitive scientific interpretation of the triptych is the danger of viewing such things as being *subjective*.

That, I think, is my task in my own collegial interactions with the supremely objective scientist that Shaikh is.

Acknowledgement

I was introduced to the classic of "*Laws of Production and Laws of Algebra: The Humbug Production Function*" (Shaikh, 1974) even before it was officially published, by Geoff C. Harcourt, in 1974, long before I could really appreciate its theoretical significance. The irony of all this resides in the fact that my own first published paper (Velupillai, 1973) was an innocently ignorant attempt at attributing priority to Wicksell on the use of the Cobb-Douglas Humbug! As for the word Humbug itself, I was fully cognizant of its meaning, having been constantly exposed to its indiscriminate use – as it appeared then, almost sixty years ago - by my Mother, in my childhood, growing up in post-colonial Colombo. In preparing this paper – and in organizing the conference - I was unselfishly, even enthusiastically, helped by my three ASSRU colleagues, Professor Stefano Zambelli, Ms. Ying-Fang Kao and Mr. Ragu Ragupathy. Naturally, none of the infelicities in this paper are anyone's responsibility but my own.

References

Bennett M., Dennett D., Hacker P., Searle J. (2003), *Neuroscience & philosophy: brain, mind and language*, Columbia University Press, New York.

Borges J. L. [1951] (1985), *Kafka and his precursors*, It. tr. Kafka e i suoi precursori, in Altre inquisizioni (1960), Feltrinelli, Milan.

Brems H. (1977), *Reality and Neoclassical theory*, Journal of Economic Theory, 15(1), 72-83.

Cohen-Cole J. (2005), *The reflexivity of cognitive science: the scientist as model of human nature*, History of the Human Sciences, 18(4), 107-139.

Goodwin R. M. (1967), *A growth cycle*, in C.H. Feinstein (ed.), *Socialism, capitalism and economic growth*, Essays Presented to Maurice Dobb, Cambridge University Press, Cambridge (UK), 54-58.

Gårdlund T. (1958), *The life of Knut Wicksell*, Almqvist & Wicksell, Stockholm.

Hicks J. R. (1960), Letter to Piero Sraffa, September, 4.

Kopmans T. C (1977), *Examples of production relations based on microdata*, in G. C. Harcourt (ed.), *The microeconomic foundations of macroeconomics*, Macmillan Press, Ltd., London, 144-171.

Levhari D. (1965), *A nonsubstitution theorem and switching of techniques*, The Quarterly Journal of Economics, 79(1), 98-105.

Myrdal G. (1930), *Vetenskap och politik i nationalekonomien*, P. A Norstedt & söner, Stockholm.

Phelps Brown E. H. (1967), The meaning of the fitted Cobb-Douglas function, The Quarterly Journal of Economics, 71(4), 546-560.

Potts R. B (1982), *Nonlinear difference equations,* Nonlinear Analysis: Theory, Methods & Applications, 6(7), 659-665.

Romer D. (2006), *Advanced macroeconomics*, (third edition), McGraw-Hill/Irwin, New York.

Samuelson P. A. (1962), *Parable and realism in capital theory: the surrogate production function*, The Review of Economic Studies, 29(2), 193-206.

Shaikh A. (1974), *Laws of production and laws of algebra: the Humbug production function*, The Review of Economics and Statistics, 56(1), 115-120.

Shaikh A. (2000), *Anwar Shaikh*, in Arestis P., Sawyer M. C. (eds.), *A biographical dictionary of dissenting economists* (second edition), Edward Elgar Publishing Ltd., Cheltenham.

Shaikh A. (2005), *Nonlinear dynamics and pseudo-production functions*, Eastern Economic Journal, 31(3), 447-466.

Shaikh A. (2010), *Reflexivity, path dependence and disequilibrium dynamics*, Journal of Post Keynesian Economics, 33(1), 3-16.

Shaikh A. (2012), *Rethinking microeconomics: a proposed reconstruction*, Working paper 06/2012, Department of Economics, The New School for Social Research, New York.

Shaikh A. (2013), *Modern political economy: real competition and turbulent macrodynamics*, Book Manuscript, The New School for Social Research.

Simon H. A. (1979), *On parsimonious explanations of production relations*, The Scandinavian Journal of Economics, 81(4), 459-474.

Simon H. A. (1997), *The failure of armchair economics*, An interview conducted by Richard. D. Bartel on September 17, 1986, in Simon H. A., *Models of bounded rationality - Empirically grounded economic reason*, Vol. 3, The MIT Press, Cambridge (MA), 387-400.

Simon H. A., Levy F. K. (1963), *A note on the Cobb-Douglas function*, The Review of Economic Studies, 30(2), 93-94.

Soros G. (2008), *The crisis & what to do about it*, The New York Review of Books, December 4, 2008, 55(19).

Sraffa P. (1926), *The laws of returns under competitive conditions*, The Economic Journal, 36(144), 535-550.

Sraffa P. (1960), *Production of commodities by means of commodities: a prelude to a critique of economic theory*, Cambridge University Press, Cambridge (UK).

Walters A. A. (1963), *Production and cost functions: an econometric survey*, Econometrica, 31(1-2) 1-66.

Wicksell K. (1900), *Om gränsproduktiviteten såsom grundval för den nationalekonomiska fördelningen*, Ekonomisk Tidskrift, 2, 305-337.

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Jayati Ghosh*

A BRIEF EMPIRICAL NOTE ON THE RECENT BEHAVIOUR OF FACTOR SHARES IN NATIONAL INCOME

Abstract

Starting from Anwar Shaikh's cogent theoretical critique of the aggregate production function, this article offers further empirical evidence on the neoclassical fallacy of the aggregate production function analysis, pointing at its analytically unsound and empirically unjustified implications. What is remarkable is that the underlying theoretical premise of the aggregate production function is still used (often without a clear explicit recognition of this) even when not just the theoretical but even the empirical basis of this is not valid (i.e. the constancy of factor shares in national income). To understand the behavior of factor shares, there is a need to look at the combination of forces that affects them, so that the conclusions arrived at by Anwar Shaikh have never appeared more valid.

JEL Classification: F43, J01, P16.

Keywords: Aggregate Production Function, Labour Income Shares, UN Global Policy Model.

Anwar Shaikh's powerful theoretical critique of the aggregate production function, first elaborated nearly four decades ago (Shaikh 1974), remains one of the most thorough demolitions of the conceptual basis of much current mainstream macroeconomic analysis. He had noted then that the so-called empirical strength of production function

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analysis is in reality nothing more than a statistical reflection of the (unexplained) constancy of income shares. Subsequently, he has further elaborated that "aggregate production functions can always be made to work on any data that exhibits roughly constant wage shares, even when the underlying technology is non neoclassical. But in so doing, they always pick up the accounting identity that underlies the data. … even when aggregate production functions appear to work at an empirical level, they provide no support for the neoclassical theory of aggregate production and distribution. On the contrary, the best of fits can utterly misrepresent the true underlying mechanisms of production, distribution, technical change, and growth" (Shaikh 2005: 462).

It is a sad commentary on the state of mainstream economics as a professional discipline, that this cogent and comprehensive critique has simply been ignored by much of the profession – including, unfortunately, those engaged in dispensing policy advice. Despite the obvious need for caution, economists continue "to do applied work with no sound foundation and dedicate some time to studying other approaches to value, distribution, employment, growth, technical progress etc., in order to understand which questions can legitimately be posed to the empirical aggregate data." (Felipe and Fisher 2003:256-7) It is not simply the understanding of how economies work that is affected. The dangers are possibly even greater when such poorly conceived analysis is then used to initiate or justify particular policy positions – in macroeconomic policies, in terms of trade liberalisation and other critical areas that affect growth, development and distribution.

What is remarkable is that the underlying theoretical premise of the aggregate production function is still used (often without clear explicit recognition of this) even when not just the theoretical but even the *empirical* basis of this (the constancy of factor shares in national income particularly for advanced capitalist economies) is not valid. Indeed, it is now evident that the period of globalisation has been one of increasing inequality, partly manifested in the decline of labour incomes as shares of the national income in many parts of the world – and often quite strongly in the rich advanced economies where they have been assumed to be quite stable.

This evidence runs counter to the perception that was widely prevalent among economists in the 20th century, that the long run process of economic growth first enhances and then reduces tendencies for greater inequality. This stemmed from the argument based on empirical observation made by Simon Kuznets in 1955, that inequality would be low at early stages of development, when societies are mostly agricultural and per capita incomes are also low. As industry develops, countries urbanize and economies grow faster, inequalities increase. Then, as countries develop further, the growing political power of lower income groups creates pressures to improve their income share and enables the introduction of broad-based policies for education and social protection.

As a consequence inequality was expected to move along an inverted U curve over time, increasing as societies develop and then decreasing. This would be reflected not only in the movement of the Gini coefficient (the standard measure of distribution of personal incomes or household consumption) but also in the functional distribution of income: the division of income between (broadly) the remuneration of workers and surplus earnings (consisting of profits, rent and interest). It was further supposed that in richer societies inequality would be relatively stable and less subject to sudden fluctuations. Indeed, in the developed countries, relatively stable shares of labour in national income had become accepted as a "stylised fact" of economic growth. This became, as Sheikh noted, the basis for the blithe acceptance of the use of aggregate production functions in macroeconomic analysis, even as concern with the functional distribution of income lagged in academic research.

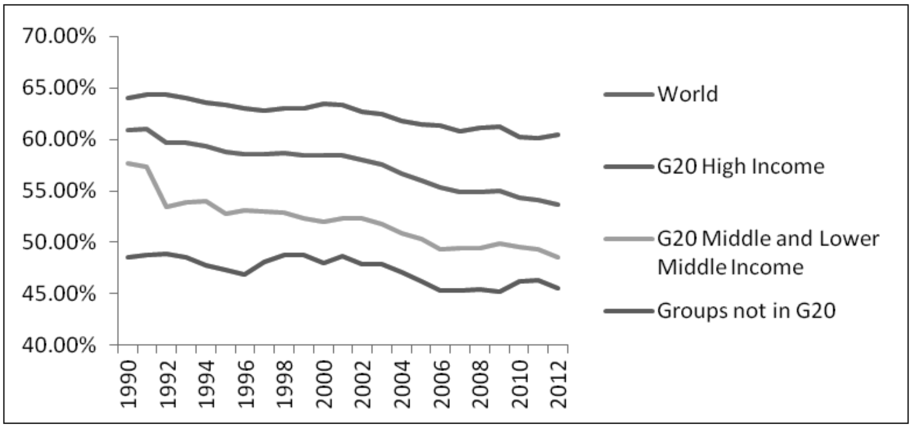
But in fact the processes of the past two decades generated rather different economic tendencies. Increasing inequality has been evident not just in rapidly growing low-income or middle income countries where the Kuznets Curve could still be used to justify it, but also in richer countries that were supposed to display more stable patterns.

The charts in Figure 1 plot the basic changes in labour shares of income in the world as a whole using data generated by the UN Global Policy Model.[1] Obviously this model only provides approximations of

[1] I am indebted to Alex Izurieta for making available the data generated by the UN Global Policy Model.

the reality, but these are based on historical data up to 2010 and model projections for the two years thereafter as well as for some intervening years.

Figure 1. Share of labour income in GDP for world and country groups.



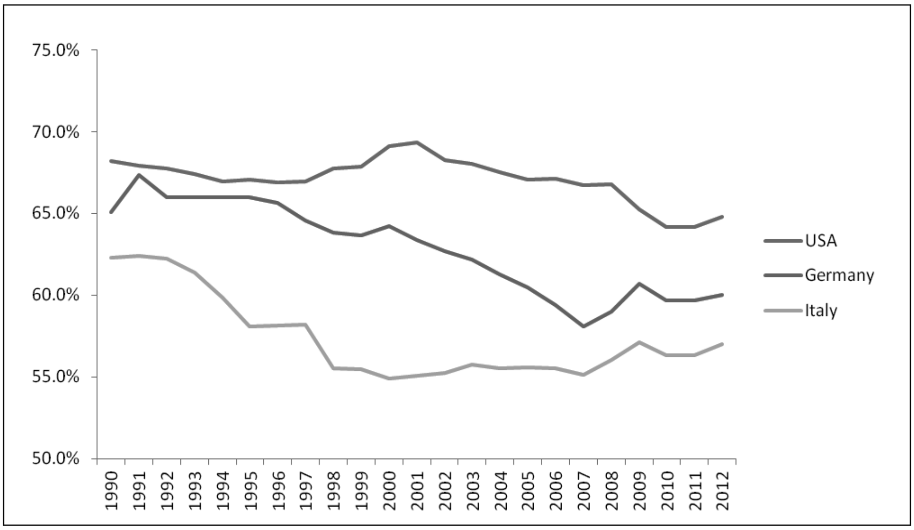
Source: elaboration from data generated by the UN Global Policy Model (2013).

The estimates refer to share of incomes from wages, salaries and the mixed income of self-employed persons, so they do contain some portion of what could be called profits especially for countries with high proportions of self-employment.

For the world as a whole, as evident from Figure 1, the share of wages and mixed incomes has been coming down continuously since 1980, and the decline has been particularly evident in the high income countries in the G20. Interestingly, the decline is also evident in the developing countries that are part of G20, although the more recent period suggests a stagnant trend around a relatively low share. Indeed, the decline in labour shares in the rich countries is possibly sharper than in the non-G20 countries that are mostly lower income countries.

In the developed world, where the income share of wage workers and self-empoyed persons has expectedly been much higher, the decline in this share has been quite marked especially since 2000.

Figure 2. Share of labour incomes in GDP in some advanced countries.

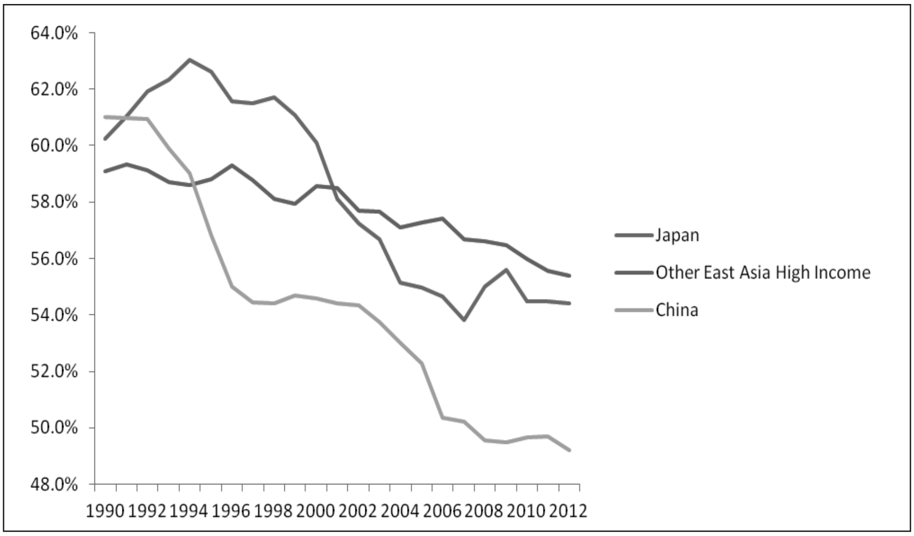


Source: elaboration from data generated by the UN Global Policy Model (2013).

Figure 2 shows this clearly for the United States, and even more so for Germany. In general, it can be observed that the most significant declines in labour shares have occurred in countries where economic strategies have been based on export-led growth, such as Germany, although they are also evident in other large and rich countries. The relationships with export orientation (and the associated competitive pressures that reduce the bargaining power of workers and therefore push down labour shares) is also evident from the pattern in Japan and other high income countries of East Asia, and of course, most of all in China.

The processes of globalisation of trade and finance that have generated increasing competitive pressures, and the associated patterns of technological change that have been labour-saving in nature, are generally accepted to explain the reduced bargaining power of workers in all of these societies, and the consequent declines in wage shares of income as well as greater instability of work. Certainly the tendencies under globalisation towards aggressive global competition and more rapid diffusion of labour-saving technological change, make for powerful forces that reduce the bargaining power of workers everywhere. This means that wage increases do not keep pace with productivity growth in most economies, and so wage shares have declined even when real wages have increased.

Figure 3. Labour income shares in some export-oriented economies.



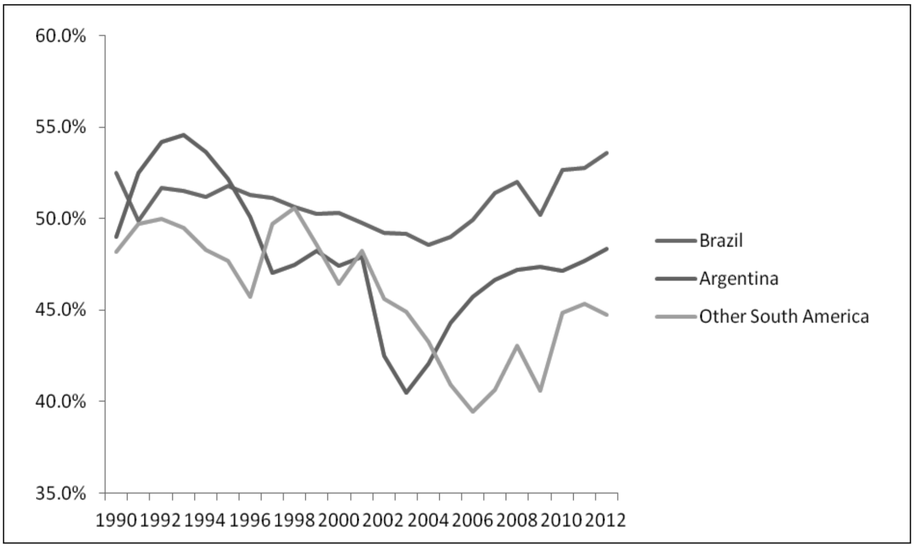
Source: elaboration from data generated by the UN Global Policy Model (2013).

However, these shifts cannot be ascribed purely to economic forces, since domestic social and political forces and policies also play important roles. In the developed industrial countries, the pressures of globalisation have not been counteracted by domestic measures that would protect and/or improve the incomes of workers; instead, it has often appeared that policies have been oriented in the opposite direction.

The pattern in China could still be used to confirm the Kuznets relationship, since China is a rapidly growing economy that still has a relatively low level of per capita income. However, such a conclusion is belied by comparative analysis. Thus, there are many developing countries (not described here) that have experienced significant declines in wage income shares in this two-decade period, without showing accelerated GDP growth. And conversely, there are other middle income developing countries that have experienced relatively rapid growth in the era of globalisation, but have still managed to reduce inequalities of income, inclduing the primary functional distribution.

The region that seems to have bucked the trend most convincingly is Latin America, which has experienced both an improvement in income inequality as measured by Gini coefficients as well as an improvement in labour shares of income in the 2000s.

Figure 4. Labour income shares in South America.



Source: elaboration from data generated by the UN Global Policy Model (2013).

As shown in Figure 4, in recent years the South American region has shown a remarkably different pattern from much of the rest of the world. The region experienced sharply declining wage shares during the "lost decade" of the 1980s, when policies of structural adjustment and fiscal repression were associated with significant increases in unemployment and informal work and declining power of workers' unions. In most countries of South America this continued into the 1990s when standard mainstream policies were still very much the norm.

But thereafter the experiences of Brazil and Argentina have shown that it is possible to have sharp increases in wage share even in a very globalised world - and incidentally this is also true of several other countries in South America (such as Ecuador and Venezuela). In Argentina labour income shares plummeted in the 1990s, and the Argentine crisis of 2001-03 caused even steeper declines. However, a combination of heterodox and worker-oriented economic policies thereafter has generated an equivalently sharp recovery in labour income shares thereafter, even though it still has not managed to bring the wage shares back to levels of the early 1990s. In Brazil, by contrast, the wage share declined more moderately in the 1990s. In Brazil, despite conservative macroeconomic policies, a combination of active labour market policies, public spending and social protection measures has led to increasing shares of labour incomes, which have even taken it to levels higher than before the "lost decade" of the 1980s.

So how could countries in Latin America buck the global trend for declining labour incomes shares even though this is a heavily globalised region? Some direct factors that are commonly noted are declines in the earnings gap between skilled and unskilled workers, in part due to the expansion of education and increase in government transfers to the poor. But these were possibly secondary factors to the more significant role of social welfare reforms, employment programmes, public spending, as well as taxation on commodity export revenues. The increase in public spending was itself the result of a shift in the economic policy model in the region that was driven by political changes in many countries, which created more social consensus that the state should serve as the engine of development, provide social welfare and be responsible for public utilities, education, including university education, health care and pensions. All of these were reflected in macroeconomic policies, taxation strategies, labour market and social protection policies and increases in social assistance.

All this reinforces the point made by Anwar Shaikh many years ago, and reiterated forcefully in his subsequent work, that aggregate production function analysis is both analytically unsound and increasingly even empirically unjustified. To understand the behaviour of factor shares, we need to look at the combination of forces that affects them: the broader forces generated by globalisation and external economic integration and the competitive pressures this results in; the concomitant processes of technological change that are also often internally generated; the political economy of the societies in which these operate; and the resultant economic policies that can dramatically affect labour incomes and their shares of national income. So the conclusions arrived at by Shaikh have never appeared more valid.

References

Felipe J., Fisher F. M. (2003), *Aggregation in production functions: what applied economists should know*, Metroeconomica, 4(2-3), 208-262.

Kuznets S. (1955), *Economic growth and income inequality*, American Economic Review, 45(1), 1-28.

Shaikh A. (1974), *Laws of production and laws of algebra: the Humbug production function*, The Review of Economics and Statistics, 56(1), 115-120.

Shaikh A. (2005), *Nonlinear dynamics and pseudo production functions*, Eastern Economic Journal, 31(3), 447-466.

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PRODUCTION FUNCTIONS AT THE BUSINESS END: THE CASE OF THE EUROPEAN FISCAL COMPACT

Abstract

Aggregate production functions are everywhere in modern economics. As a theoretical tool they have severe limitations. When used for practical purposes by practical people who do not understand the theoretical niceties of the approach, the aggregate production function ceases to be an intellectual curiosum, and instead becomes something altogether more dangerous. Shaikh (1974) has demonstrated the aggregate production function can be used to justify any policy, and so can lead to perhaps unnecessary real world hardship via this misuse. In this paper we document one such potential misuse, as European authorities enact the provisions of the Treaty on Stability, Coordination and Governance in the Economic and Monetary Union, hereafter simply called the Fiscal Compact. In this paper for a range of EU and OECD countries we simulate a range of functional forms of the production function, and show how, for each functional form and estimation method, a qualitatively different *policy outcome is inferred for each country. Economic policy at the super-national level should not rely on such flimsy theoretical and empirical foundations. We conclude that when production functions are used at the business end, they may be damaging to the business of policy making itself.*

JEL Classification: D24, E23, E30.

Keywords: Production Function; European Union; Fiscal Compact.

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1. Introduction

The goal of this paper is to show that as one varies the functional form of an aggregate production function, one obtains qualitatively different policy prescriptions for the same underlying time series data.

This is not a trivial point, as aggregate production function estimation forms the basis of policy evaluation and deployment in developed countries, especially when production functions are used to calculate the potential output of an economy.

In particular, we study the variability of policy prescriptions coming from alternative specifications of the aggregate production function as it is applied to measurement of output gaps and structural deficits.

We study five functional forms and 18 OECD countries and use the same estimation techniques as the European Commission (Roeger, 2006).

Our paper is motivated by the interlocking national and international crises in Europe, and the set of policy responses to these crises.

In response to the sovereign debt crisis sweeping Europe, the nations of the Eurozone have agreed to strengthen fiscal discipline. One part of this fiscal discipline is the intergovernmental Treaty on Stability, Coordination and Governance in the Economic and Monetary Union, hereafter called the Fiscal Compact. The form this fiscal discipline between member States will take is enhanced macroeconomic oversight of one another's budgets, and writing cyclically adjusted balance budget provisions into law.

Importantly, these cyclically adjusted estimates will be produced using a production function method. We aim to show empirically this method is questionable.

The Fiscal Compact is one step towards German-style fiscal rectitude for the Eurozone. Therefore, at the 'business end' of macroeconomic policy, we will describe the difficulties inherent in using production-function based quantities to guide policy at the national and European level.

The rest of this paper is laid out as follows. Section 2 describes the economics of the fiscal compact. Section 3 discusses theoretical issues with the production function approach. Section 4 describes our results. Section 5 concludes.

2. The Fiscal Compact

Europe's Treaty on Stability, Coordination and Governance in the Economic and Monetary Union (the *fiscal compact*) is based on the notion of the cyclically adjusted structural deficit. In order to allow for counter-cyclical measurements, the output gap is decomposed into two parts: a cyclical component and a structural component. Each government is authorized to have a budget deficit equal to the cyclical component of the output gap. Every euro of deficit above that limit is considered a structural deficit.

Signatories to the Treaty agree to rigourosly implement several simple fiscal rules that have existed since the 1996 Stability and Growth Pact was signed into EU law.[1]

If a country has a budget deficit of more than 3% for more than 3 years, this is considered an excessive deficit. An excessive deficit procedure commits member States to implement structural reforms (for example cutting government expenditure, increasing taxes, reducing minimum wages, freeing up protected sectors) to reduce this deficit. The European Commission will monitor all member States using an alert mechanism report which measures internal imbalances within a country like changes in house prices and its, general government debt, and external imbalances like changes in its export market share and labour costs.

In the event of a persistent imbalance, the member State must reduce the gap between the current level and the 60% threshold by 1/20th per year. A legally binding structural deficit limit of 0.5% of GDP will be also introduced. Importantly, for countries below the 60% debt threshold, the limit will be 1%.

The measurement of structural balances relies on the notions of potential output and output gaps. Potential output in an unobservable variable which represents:

> . . . the maximum level of durably sustainable production, without tensions in the economy, and more precisely without acceleration of inflation (Ladiray et al., 2003, page 1).

[1] The fiscal rules were (and are) that: a country's budget deficit can not be more than 3% of Gross Domestic Product (GDP); a country's public debt, can not exceed 60% of GDP; and a country's budget must remain in balance over a 3 year term.

The output gap is defined as the difference between potential output and realised output. Once the output gap is measured, it is then possible to compute the structural deficit of a country using the following simple rule: government deficit minus a fraction of the output gap.[2] The idea behind this formula is that during a business cycle the state should be able to develop counter-cyclical policies. However, only part of the output gap is due to the business cycle while the rest of the gap between potential and realised output is caused by structural issues and should be addressed by structural policies rather than counter-cyclical measures.

There are two main procedures to compute the unobservable potential output: *statistical* methods and the *production function* approach.

Statistical methods rely on fitting trends using filters with the help of a model, usually something like a structural VAR or a semi-structured method like the HP filter, the Structural Time-series Model, (STM) or Unobserved Components (UC) approaches. Apel and Jansson (1999) provides an excellent summary.

The production function approach relies upon production functions as simple as the textbook Cobb-Douglas (Roeger, 2006). This paper treats only the case of the production function approach.

Groups like the OECD and IMF use a statistical approach, while the European Commission uses a production function approach that relies on a natural rate of unemployment calculation, which we detail below.

3. Production functions: theory and empirics

The semantics of the production function were analysed by Cohen and Harcourt (2003). Cobb and Douglas themselves describe them in the following way. In order to find the estimated production function, we pretend there is a production process, P , at work somewhere in the real world. We would like to define the best representation P' of that production process given available data.

We define an aggregate production function as a function estimated using constant price value data. The inputs to this function are, in

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[2] The fraction of the output gap which is seen as cyclical is different for each country.

theory, physical quantities of labour and capital and eventually total factor productivity which captures technology. The present section will describe one the most famous functional form of an aggregate production function, that is the Cobb-Douglas production function (Cobb and Douglas, 1928).

3.1 The Cobb-Douglas production function and other functional forms

The Cobb Douglas production function has been widely used in the economic literature for two main reasons. First, it is based on the neoclassical theory of wages and prices.

The exponent parameter represents the wage share and the profit share, respectively defined as the marginal productivity of labor and capital. (Cobb and Douglas, 1928, page 151) write:

> to say that P' represents the actual production P is to give particular expression to a well-known theory.

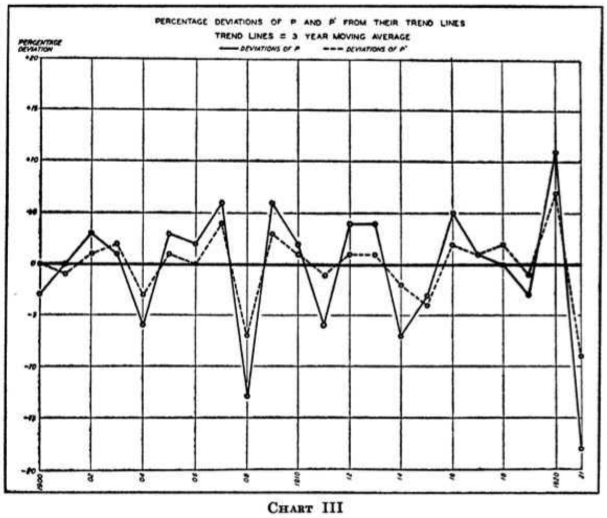
The 'well-known theory' is, of course, neoclassical economics.

Second, it seems from the outset that the functional form has been empirically proven. Cobb and Douglas (1928) found that after constructing indices of production, from 1899 to 1922 the functional form $P' = 1.01L^{3/4}C^{1/4}$ was a good fit (see fig. 1 and the subsequent empirical work by Zellner et al. (1966) and criticism by Cotis et al. (2003)).

Cobb and Douglas couched their results carefully in their initial and subsequent papers, though in later writings Douglas admits his colleagues at Chicago did not think much of the approach at the time (Douglas, 1976). Despite these caveats, for most estimations the goodness of fit usually lies between 0.9 and 1.

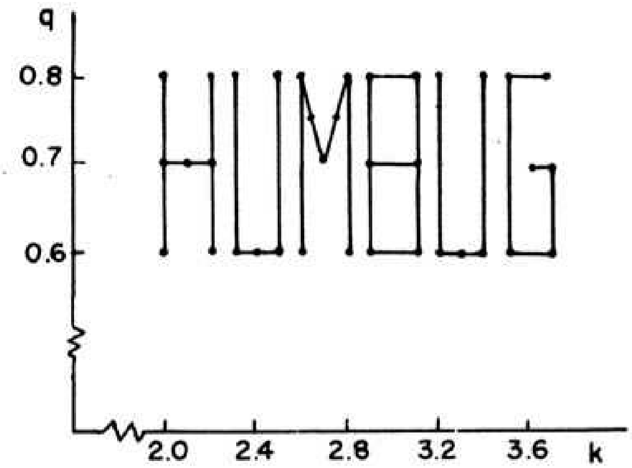
In a classic paper, Shaikh (1974) elegantly demonstrated that this second property of the Cobb-Douglas production function is a purely mathematical construct.

Figure 1: Cobb and Douglas (1928) fitted curves, too good to be true?



Source: Cobb and Douglas (1928).

Figure 2: Humbug production function, still empirically valid with an R^2 of 0.9964.



Source: Shaikh (1974).

Indeed, Shaikh showed that even with a 'humbug' production function (see Figure 2), the goodness of fit for the Cobb Douglas is 0.9964.

We will now examine Shaikh's insights in the light of the recent Fiscal Compact, using a range of functional forms, described below, and OECD data for several developed countries.

3.1.1 Functional form

Using five different functional forms for production functions, and the European Commission's estimation method, we will analyse the impact of each functional form on the computed output gap for a range of developed countries and the resulting estimated structural deficits. We estimate:

- Cobb-Douglas(CD): $Y = AK^{\alpha}L^{1-\alpha}$
- Constant Elasticity of Substitution (CES): $Y = A\left[\alpha K^{\gamma} + (1-\alpha)L^{\gamma}\right]^{\frac{1}{\gamma}}$
- Translog: $\ln(Y) = \alpha_1 \ln(K) + \alpha_2 \ln(L) + \beta_{11} \ln(K^2) + \beta_{12} \ln(K)\ln(L) + \beta_{22} \ln(L^2)$,
- Polynomial (cubic): $Y = \alpha_1 K + \alpha_2 K^2 + \alpha_3 K^3 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3$
- Leontief: $Y = \min[\alpha K, \beta L]$

All these functional forms are based on a combination of capital K, labor L, possibly a constant term A, and a number of parameters α_i, β_{ij}, γ, as required. In order to be able to compute potential output, one needs to obtain time series for each of the variables K, L and A and then compute the parameters, usually via an ordinary least squares method, as all the functional forms are linear, quasi-linear, or log-linear.

3.2 Data

In order to construct the different time series, we use the OECD data base for all countries OECD. The time range is from 1950 to 2012. However, we have complete data sets (i.e. observations for all variables) only between 1990 and 2011.[3]

As the time series for each variable of the production function is not readily obtainable, we construct them. The next section will describe how it is usually done.[4] One further note, whenever a filtering technique is used,[5] may it be Kalman or Hodrick-Prescott, we have decided to start the filter only 10 periods after the beginning of the observations, in order to avoid as much as possible issues related to end-of-data weights.

3.3 European Commission production function based potential output

The European Commission (EC) estimates potential output according to the data used in figure 3. All production function are based on at least two time series: capital input K and labour input L. Furthermore, the CES based functional forms (the Cobb-Douglas and the CES) require the so-called total factor productivity (TFP). All these time series need to be build as they are not directly observable.[6] The present section will present how these series are build and what are the issue rising from the assumption used.

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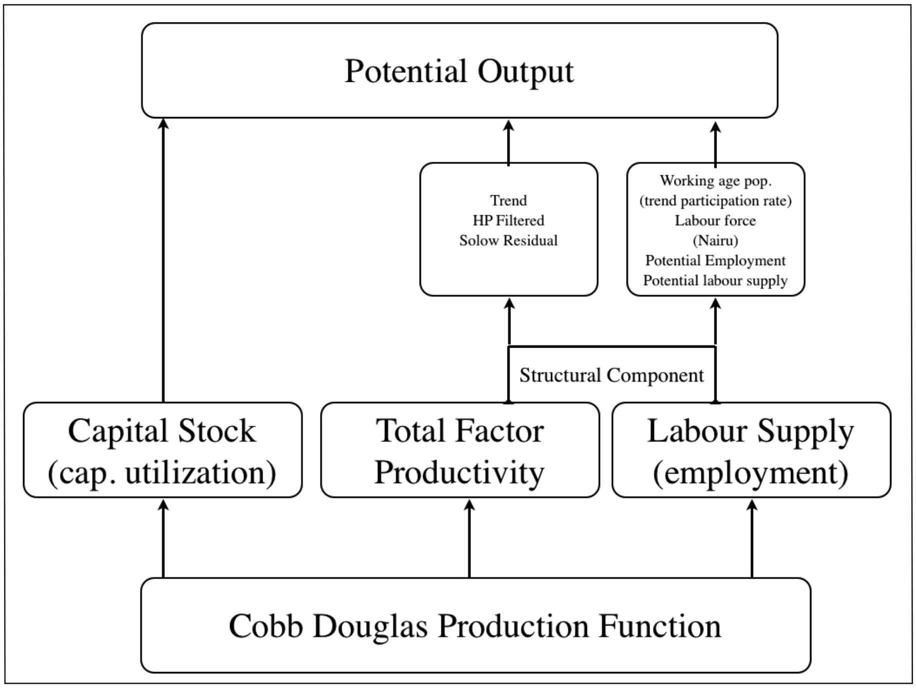
[3] However, some countries do not have that full range of data, some time series may start as late as 1995 and other might end as early as 2008. All countries have at least 15 years of complete data sets.

[4] Appendix B contains a description of each time series used, in which OECD table it can be found and what are its units.

[5] Appendix A contains a quick aide-mémoire of the different filtering technique.

[6] Recall that these are highly aggregated time series. The aggregation of capital is a major issue as it is difficult, if not impossible, to aggregate capital from a manufacture sector, say steel industry, and a service sector, say IT Cohen and Harcourt (2003). This is indeed one of the major statistical weakness of production function based potential output Claus et al. (2000).

Figure 3: Estimation flow for potential output calculation using Cobb-Douglas production function.



Source: elaboration adapted from Roeger (2006).

3.3.1 Capital stock

The capital stock time serie is a tricky index: it is as summing pears (say computers) and apples (say cold rolling mills) and is usually not found in most macroeconomic database. This why the index is usually build via the perpetual inventory assumption and two time serie: gross capital formation (*GKF*) and GDP growth (*GrGDP*).

The perpetual inventory assumption uses the idea that the capital stock at the end of time t is passed onwards to time $t + 1$ in order to produce. We thus have that the capital stock at time $t + 1$ is equal to the capital stock at time t minus depreciation, plus investments (1). From this equation and assuming a constant depreciation rate ($\delta_t = \delta$), it is possible to compute K_t, having an initial capital stock K_0 (2).

$$K_{t+1} = K_t \cdot (1 - \delta_t) + I_t \tag{1}$$

$$K_t = (1 - \delta)^t \cdot K_0 + \sum_{i=1}^{t-i} (1 - \delta)^{t-i} \cdot I_i \tag{2}$$

However, as we do not have the investment time series from year 0, we need a further hypothesis, that is we assume that the country is at its steady state capital-output ratio (3), where $k = K/Y$ is the capital-output ratio, $i = I/Y$ is the investment rate and g is the GDP growth rate. From (3), it is easy to infer the first value of the capital stock time series (4), assuming that the growth rate of GDP is equal to the average growth rate observed and that the depreciation rate is equal to 5%.[7]

We can then construct the whole time series using (1).

$$k = \frac{i}{g + \delta} \tag{3}$$

$$K_{1990} = \frac{GKF_{1990}}{\overline{GrGDP} + 0.05} \tag{4}$$

Usually, potential capital stock is set equal to the current one. The reason behind that assumption is that the full capital stock is potentially available for use (Fuentes et al., 2007) or that there are no heavy fluctuation in capital stock (Dimitz, 2001). In that case, potential output will be computed given the capital stock.

However, this measure of potential output does not account for the business cycle in capital stock since capital stock formation is likely be large or small according to the different phases of the business cycle. Hence, we use a second time series, potential capital, where we compute the trend of the gross capital formation in order to avoid to lose capital formation cyclicality.

As for any HP filter, the length of the time serie is critical and the extremities of the data have higher weight than central values. We thus compute two different measures for potential capital, based on (1). The first measure

[7] We assume a common 5% depreciation rate, having no information on country-specific depreciation rates

$$K_t^{p,1} = K_{t-1}^{p,1} \cdot (1-\delta) + GKF_{t-1}^{p} \tag{5}$$

$$K_t^{p,2} = HP(\mathbf{K}_{t-1}) \tag{6}$$

In order to test for the rationality of assuming potential capital equal to observed capital, we run normality tests on the vector containing the differences between potential capital (the two forms) and observed capital. Indeed, if potential capital was equal to observed capital, then the two series should not be significantly different and thus the difference between the two series should be of zero mean (and ideally normally distributed). As table 1 shows, almost all countries show p-values below the critical value of 0,05 for both the Shapiro-Wilk and the Lilliefoers normality tests.

Table 1: Normality tests p-value and mean for both potential capital forms.

	$K^p = K^p_{-1} \cdot (1-\delta) + GKF^p_{-1}$			$K^p = HP(\mathbf{K}_{-1})$		
Country	Mean	Shapiro-Wilk	Lilliefoers	Mean	Shapiro-Wilk	Lilliefoers
Australia	-1,703	0,146	0,020	-2,139	0,064	0,051
Austria	-0,011	0,003	0,001	-0,517	0,001	0,008
Belgium	-0,091	0,017	0,001	-0,507	2,35e-004	0,001
Canada	-0,246	0,036	0,012	-1,015	0,012	0,049
Czech Republic	-0,101	0,009	0,002	-0,349	0,024	0,001
Denmark	-0,589	0,055	0,087	-0,920	0,008	0,055
Finland	-0,566	0,035	0,008	-0,960	0,080	0,081
France	-1,445	0,110	0,015	-1,857	0,004	0,006
Germany	0,373	0,245	0,014	-0,236	4,88e-004	8,82e-005
Hungary	1,062	1,43e-004	2,10e-004	-0,655	0,014	9,12e-005
Ireland*	-0,062	0,310	0,095	-1,633	0,059	0,036
Italy	-0,823	0,010	0,012	-1,674	7,15e-006	7,29e-007
Luxembourg	-1,380	0,034	0,004	-1,705	0,009	0,013
Netherlands	0,128	0,020	0,008	-0,478	0,038	2,07E-004
Norway	-1,284	0,075	0,024	-1,304	0,065	0,021
Portugal	-1,938	8,15e-005	5,12e-006	-3,007	2,17e-005	2,30e-004
Spain	-1,369	0,228	0,065	-2,071	0,137	0,010
United Kingdom	-0,197	0,050	0,001	-1,164	0,006	0,026

Source: elaboration adapted from Roeger

Note that only Ireland has both positive normality tests (p-value >0,05) and a mean close to 0, for the first formulation of potential capital. No countries has both positive tests and 0 mean for the second formulation of potential capital.

Furthermore, we observe that the mean show values significantly different from 0 in most cases. Only Ireland has both positive normality tests and a mean close to 0. The conclusion of this section is that if one wants to compute the output gap based on a production function, she should account for the capital gap arising from under or over investment and not assume the potential capital to be equal to the current level of capital. Furthermore, the potential capital stock should be computed using potential gross capital formation.

3.3.2 Labor

The labour time serie needs more computational steps. Labour is usually computed as total hours worked during the time period. Total potential hours worked is computed using (7) where pop_t is total population, $part_t$ is the participation rate, $nairu_t$ is the non-accelerating inflation rate of unemployment and $hours_t$ is average hours worked per capita.

$$L_t = pop_t.part_t(1 - nairu_t)hours_t \tag{7}$$

All these time series are directly obtained from statistical databases except for the nairu. In order to compute the nairu, a Kalman filter is used, based on a Philipps curve (8), where w_t is the log of nominal wages, pr_t is the log of labour productivity, ws_t is the log of the wage share, u_t is the unemployment rate. In this case, the measurement equation of the Kalman filter is given by the Philips curve (8) and the transition equation is given by (9).

$$\Delta w_t = \phi^{pr}\Delta pr_t + \phi^{ws}\Delta ws_t - \beta(u_t - nairu_t) + \epsilon_t^w \tag{8}$$

$$nairu_t = \alpha.nairu_{t-1} + \epsilon_t^{nairu} \tag{9}$$

Once all time series in (7) are de-trended via an HP filter, it is possible to compute potential labour input. That is maximum labour input such that inflation remain low.[8]

8 Note that this nexus between potential labour input (and thus potential output) and low inflation is implicit since potential labour input is based on the nairu. This justifies why central banks are computing potential output: in order to detect risks of inflation (Claus et al., 2000).

3.3.3 Bringing it together

If needed, the total factor productivity (TFP) values are computed as the HP-filtered Solow residual. Since the TFP is computed as a residual, its value depends on how capital and labor are computed. Furthermore, we face here some theoretical choices. Indeed, the Cobb-Douglas function is widely used because its parameter represent the income share of capital and labor. One could thus be tempted to avoid the regression step and directly use each country average wage and profit shares to compute the TFP, or even use common values for all countries. On the other hand, when running a regression, the modeller has to chose to let the parameters being freely determined or to constrained them so that their sum is equal to one. We decided to use all four configurations and to compare the results. Formally, we thus have four specifications for the Cobb-Douglas production function, and then one for each of the other functional forms. Table 2 contains a description of each specification.

Table 2: Functional form specifications.

Name	Description
CD-1	Cobb-Douglas, parameters not constrained: $Y = A \cdot K^{\alpha} \cdot L^{\beta}$
CD-2	Cobb-Douglas, parameters constrained: $Y = A \cdot K^{\alpha} \cdot L^{1-\alpha}$
CD-3	Cobb-Douglas, parameters equal to average income share per country: $Y = A \cdot K^{(1-WS)} \cdot L^{WS}$
CD-4	Cobb-Douglas, parameters equal to common average income share: $Y = A \cdot K^{0.37} \cdot L^{0.63}$
CES	Constant Elasticity of Substitution: $Y = A[(1-WS)K^{1/6} + (WS)L^{1/6}]^6$
TRL	Translog: $\ln(Y) = \alpha_1 \ln(K) + \alpha_2 \ln(L) + \beta_{11} \ln(K^2) + \beta_{12} \ln(K)\ln(L) + \beta_{22} \ln(L^2)$
POL	Polynomial (cubic): $Y = \alpha_1 K + \alpha_2 K^2 + \alpha_3 K^3 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3$
L	Leontief: $Y = \min[\alpha K, \beta L]$

Source: our elaboration.

3.3.4 Cobb-Douglas specifications

Table 3 shows the results for the estimations of the Cobb-Douglas parameters. The first point to note is that the sum of the parameters, for the unconstrained specification, is seldom close to one (Belgium, Denmark, Germany, Luxembourg, Portugal and the United States being exceptions) and sometimes even negative. Furthermore, while the statistical significance of the capital parameter is always good, statistical significance for the labour parameter is rather mitigated.

Furthermore the values obtained range from largely negative (Czech Republic) to null (France) to largely positive (Finland and Ireland). This first statistical estimation casts some doubt on the pertinence of using a unconstrained Cobb-Douglas production function, even if the R^2 values are good (or, perhaps even too good).

The second specification consists in constraining the parameters and have their sum equal to one. Bear in mind that this implies an additional theoretical choice. The second part of table 3 shows the statistical results for the estimation of this specification.

Table 3: Cobb-Douglas estimations.

	$Y = A \cdot K^{\alpha} \cdot L^{\beta}$				$Y = A \cdot K^{\alpha} \cdot L^{1-\alpha}$		
Country	α	β	Σ	R^2	α	P. Share	R^2
Australia	0,774 (0,057) ***	0,506 (0,301)	1,280	0,996	0,837 (0,015) ***	0,367	0,994
Austria	0,734 (0,021) ***	0,981 (0,555)	1,715	0,988	0,731 (0,022) ***	0,197	0,987
Belgium	0,650 (0,014) ***	0,418 (0,151) *	1,068	0,995	0,654 (0,011) ***	0,301	0,995
Canada	0,670 (0,072) ***	0,773 (0,377)	1,444	0,995	0,771 (0,018) ***	0,360	0,991
Czech Republic	0,326 (0,048) ***	-3,133 (0,859) **	-2,807	0,953	0,495 (0,040) ***	0,430	0,912
Denmark	0,670 (0,010) ***	0,413 (0,164) *	1,082	0,996	0,670 (0,010) ***	0,301	0,996
Finland	0,847 (0,031) ***	1,089 (0,414) *	1,935	0,975	0,832 (0,034) ***	0,307	0,968
France	0,649 (0,018) ***	0,001 (0,274)	0,649	0,991	0,663 (0,014) ***	0,239	0,992
Germany	0,613 (0,021) ***	0,469 (0,112) ***	1,082	0,990	0,602 (0,011) ***	0,281	0,994
Hungary	0,629 (0,015) ***	1,391 (0,434) **	2,020	0,993	0,611 (0,015) ***	0,361	0,991
Ireland	0,554 (0,060) ***	1,809 (0,402) ***	2,363	0,977	0,751 (0,042) ***	0,338	0,941
Italy	0,571 (0,016) ***	-0,161 (0,257)	0,410	0,989	0,588 (0,015) ***	0,256	0,987
Luxembourg	0,785 (0,131) ***	0,136 (0,388)	0,921	0,991	0,746 (0,027) ***	0,416	0,975
Netherlands	0,780 (0,054) ***	0,529 (0,260)	1,309	0,994	0,853 (0,020) ***	0,291	0,990
Norway	1,056 (0,059) ***	-0,621 (0,464)	0,435	0,980	1,002 (0,043) ***	0,373	0,973
Portugal	0,547 (0,008) ***	0,512 (0,088) ***	1,059	0,997	0,547 (0,008) ***	0,316	0,997
Spain	0,517 (0,030) ***	0,914 (0,159) ***	1,431	0,995	0,602 (0,021) ***	0,313	0,983
United Kingdom	0,729 (0,008) ***	0,597 (0,168) **	1,326	0,998	0,732 (0,008) ***	0,302	0,998

Source: our elaboration from OECD data (2012)

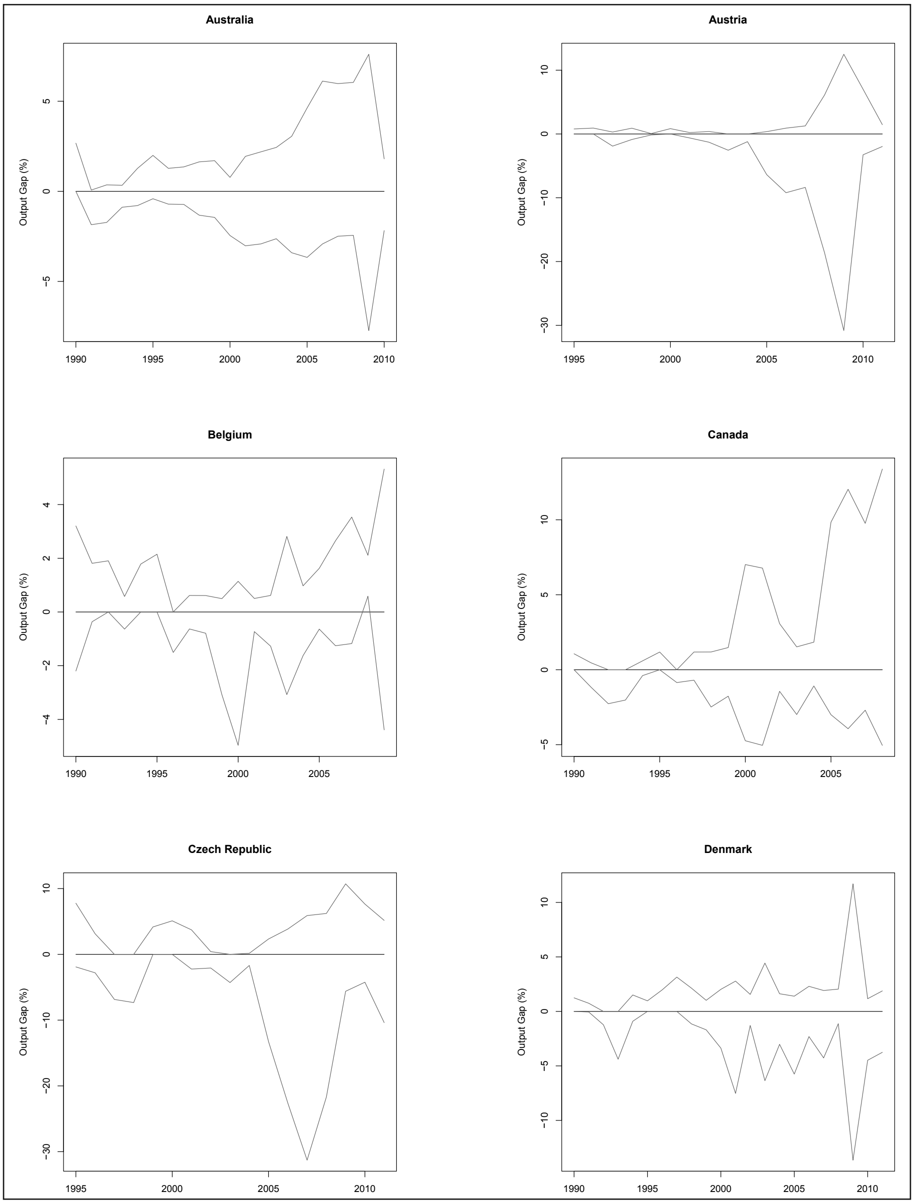
We observe that all results are statistically significant and that the R^2 results are also very good for all regressions. However, when comparing the statistical result (α) with its theoretical counterpart (the profit share, P. share in the table), we observe that the theoretical motivation, that is the Cobb-Douglas parameters representing the income share of each component, does not tally with the statistical results. Something important is amiss.

4. Alternative measurements

Following the methodology explained in the previous section, we have computed 16 different potential output (2 per specification defined in table 2, one for observed capital stock and one for potential capital stock). We then computed 16 different output gaps per country. Figures 4, 5 and 6, show the bandwidth of these 16 output gaps (that is minimum and maximum values computed per year per country). We observe that for all countries and for each year, we observe positive and negative output gaps, depending on the way these output gaps are computed.

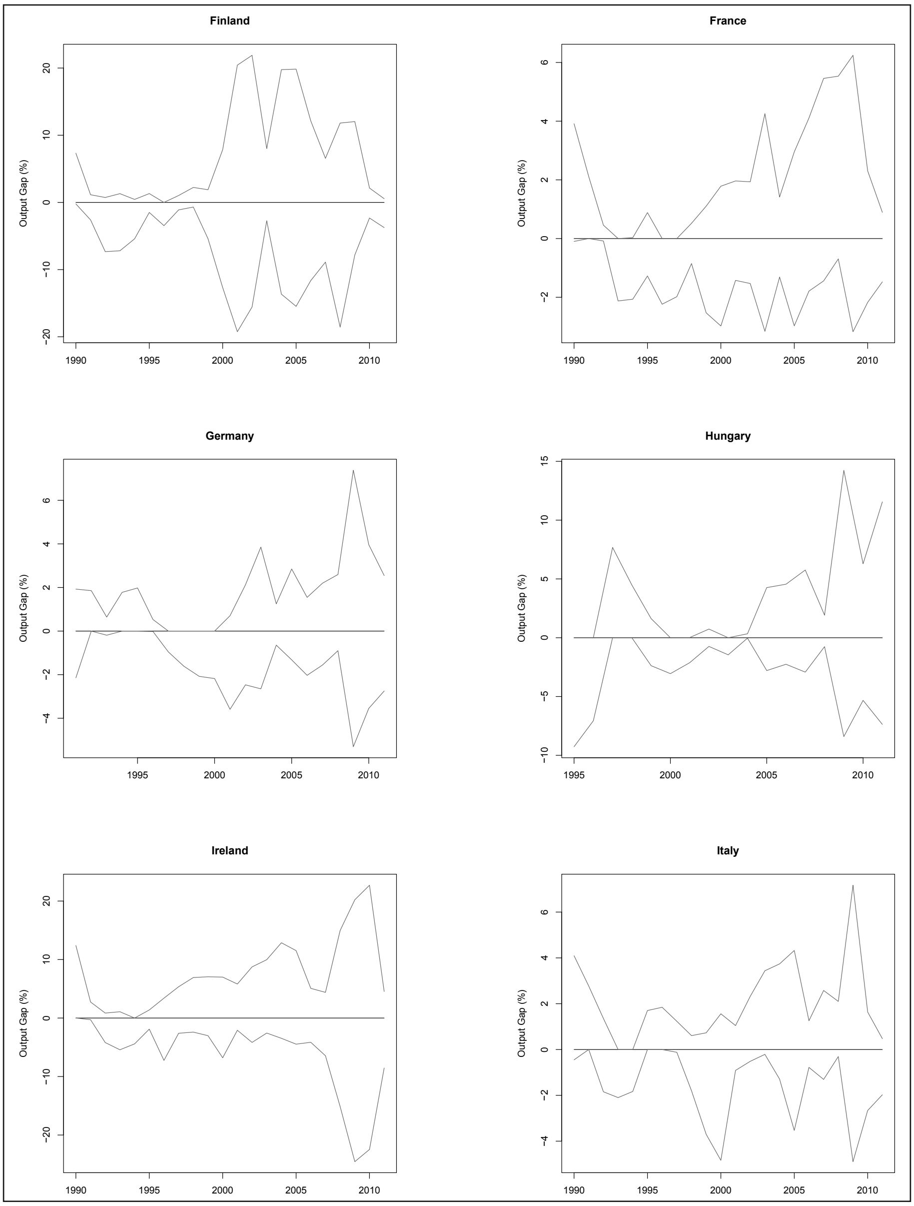
Once the output gaps are computed, it is possible to determine what part of the public deficit is considered as structural (using a common 40% of the output gap) and then determine what should be the reduction in government expenditure in order to comply the golden rule of 0.5% structural deficit. Figures 7, 8 and 9 show the bandwidth of reductions requested, depending on the functional form specification used to compute the output gap.

Figure 4: Output gap bandwidth per country.



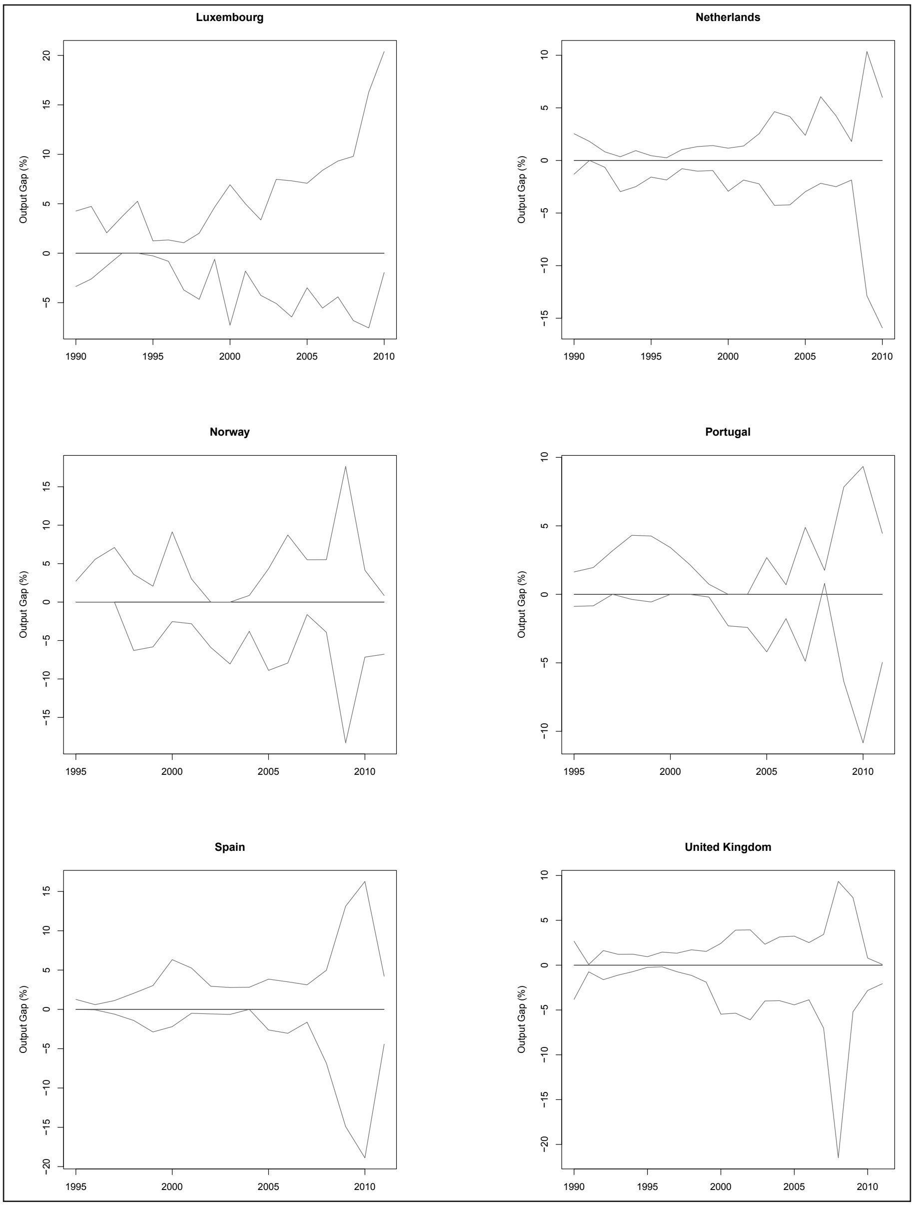
Source: our elaboration from OECD data (2012).

Figure 5: Output gap bandwidth per country.



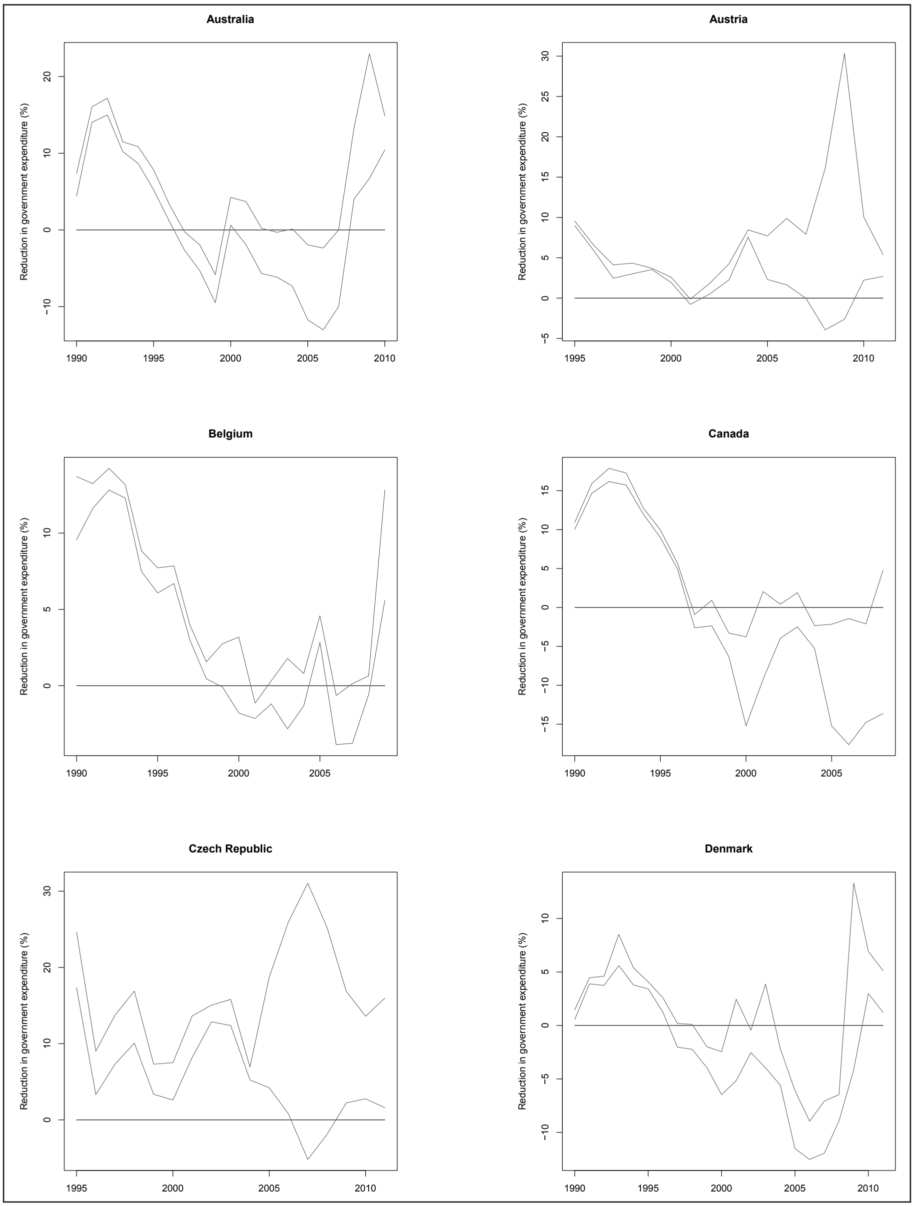
Source: our elaboration from OECD data (2012).

Figure 6: Output gap bandwidth per country.



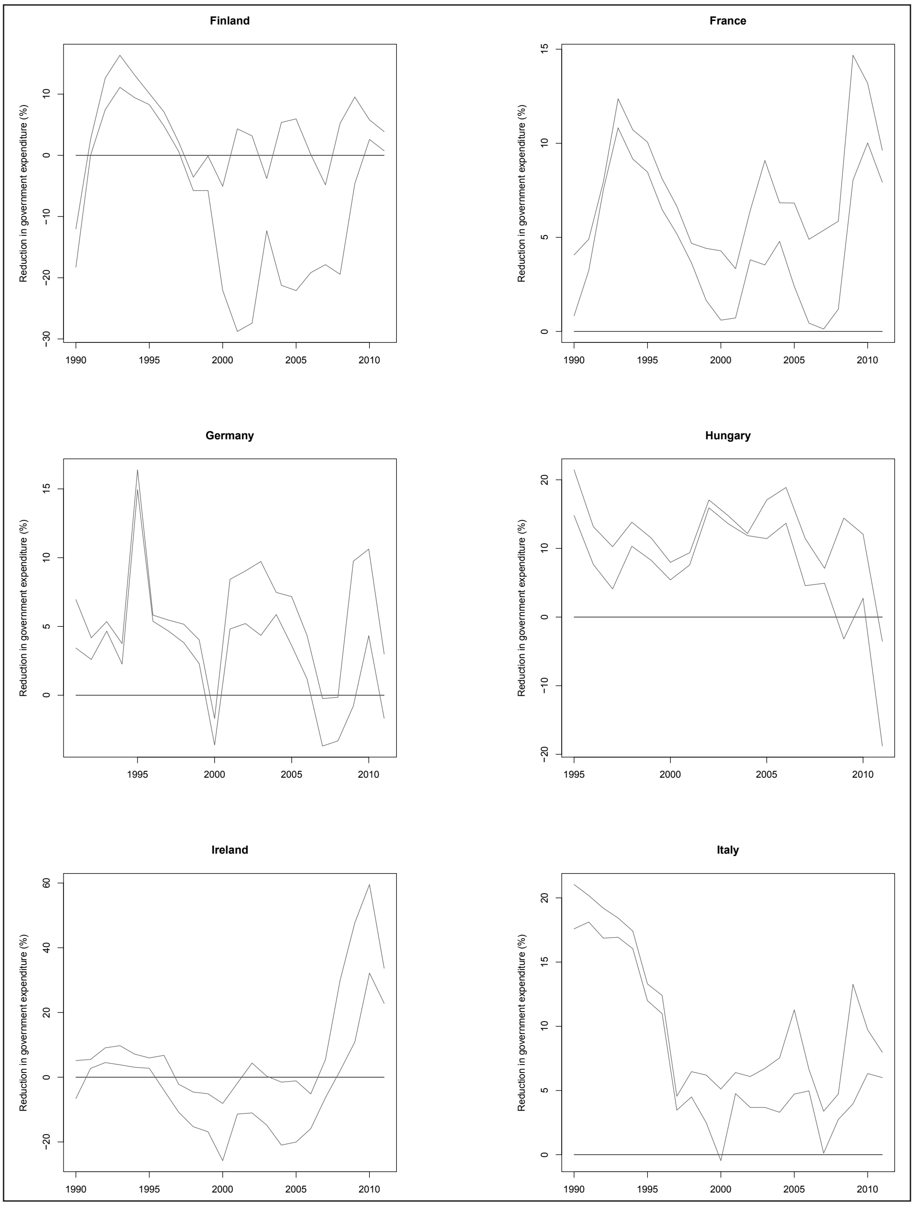
Source: our elaboration from OECD data (2012).

Figure 7: Requested government expenditure reduction per country.



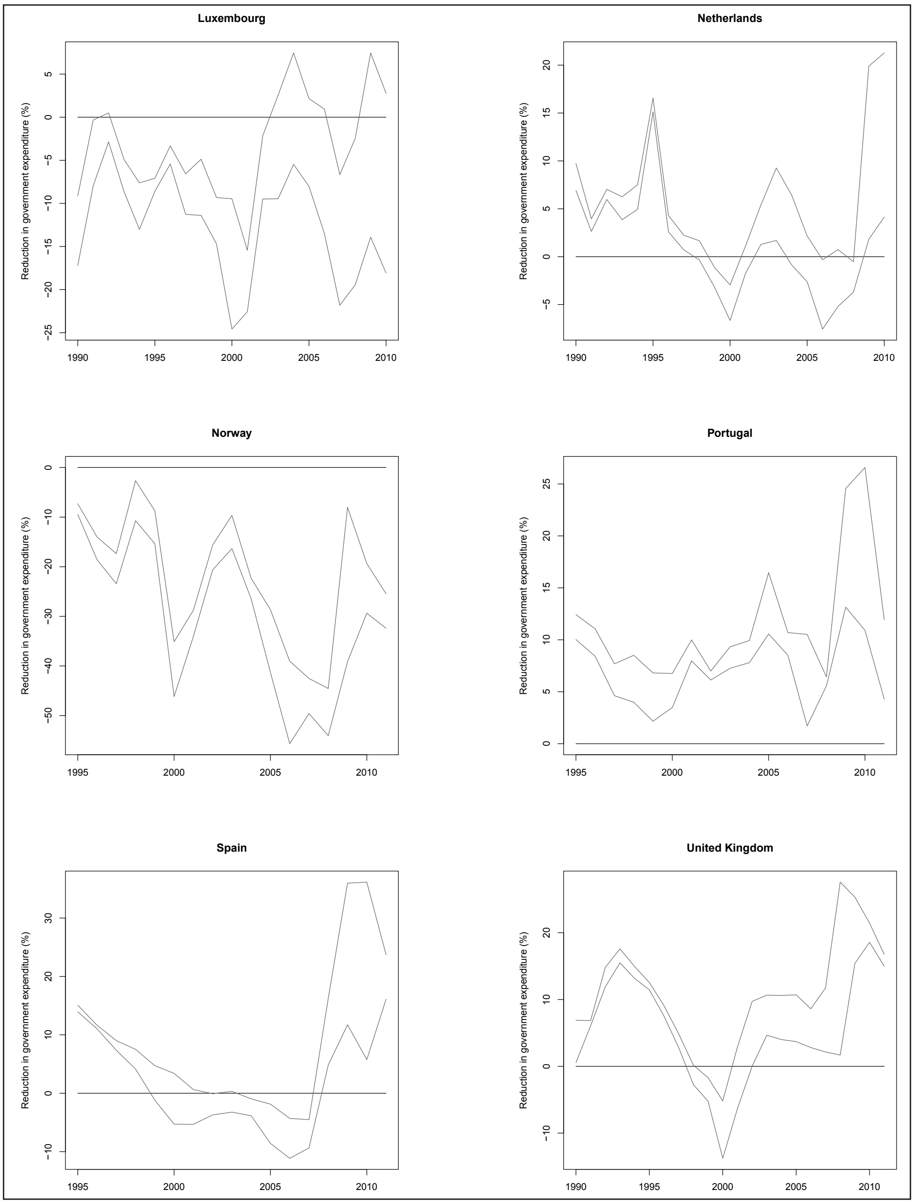
Source: our elaboration from OECD data (2012).

Figure 8: Requested government expenditure reduction per country.



Source: our elaboration from OECD data (2012).

Figure 9: Requested government expenditure reduction per country.



Source: our elaboration from OECD data (2012).

5. Conclusion: theoretical rubber meets empirical road: The case of the humbug production function and the fiscal compact.

This paper describes a simple experiment: we replicate the output gap estimation methodology of the European Commission for a panel of OECD countries from 1990 to 2012. We vary the functional form of the production function used, and we show that for a range of plausible estimated and parameter values, the policy prescriptions of the Commission's methodology would be widely divergent.

So what? It should be obvious that an alternative functional form would yield different outputs. We agree. What is striking is not that the outputs are different, it is *how* qualitatively different these outputs are. In 1974 Anwar Shaikh showed us Cobb Douglas production functions were analytically useless for practical purposes. This paper extends, and deepens that critique by applying the same reasoning to alternate functional forms. The Fiscal Compact has effectively written the Cobb Douglas into the constitutions of many European States. At the business end, this is bad, and potentially dangerous, policy.

Acknowledgement

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References

Apel M., Jansson P. (1999), *System estimates of potential output and the NAIRU*, Empirical Economics, 24(3), 373-388.

Claus I., Conway P., Scott A. (2000), *The output gap: measurement, comparisons and assessment*, Research paper no. 44, Reserve Bank of New Zealand.

Cobb C. W., Douglas P. H. (1928), *A theory of production*, American Economic Review, 18(1), 139-165.

Cohen A. J., Harcourt G. C. (2003), *Retrospectives: whatever happened to the Cambridge capital theory controversies?* The Journal of Economic Perspectives, 17(1), 199-214.

Cotis J.-P., Elmeskov J., Mourougane A. (2003), *Estimates of potential output: benefits and pitfalls from a policy perspective*, OECD Working paper, retrieved at http://www.oecd.org/dataoecd/60/12/23527966.pdf on August 30, 2013, also in Reichlin L. (ed.) (2005), *Euro area business cycle: stylised facts and measurement issues*, CEPR, London.

Dimitz M. A. (2001), *Output gaps in european monetary union new insights from input augmentation in the technological progress*, Reihe ökonomie economics series, Institute for Advanced Studies, Vienna.

Douglas P. H. (1976), *The Cobb-Douglas production function once again: its history, its testing, and some new empirical values*, The Journal of Political Economy, 84(5), 903-915.

Fuentes R., Gredig F., Larraín M. (2007), *Estimating the output gap for Chile*, Working papers Central Bank of Chile, No. 455.

Hamilton J. D. (1994), *Time series analysis*, Princeton University Press, Princeton.

Hodrick R. J., Prescott E. C. (1997), *Postwar U.S. business cycles: an empirical investigation*, Journal of Money, Credit and Banking, 29(1), 1-16.

Kalman R. E. (1960), *A new approach to linear filtering and prediction problems*, Journal of Basic Engineering, Transaction of the ASME, D(82), 35-45.

Kalman R. E. (1963), *New methods in wiener filtering theory*, in Bogdanoff J. L., Kozin, F. (eds.), Proceedings of the First Symposium of Engineering Application of Random Function Theory and Probability, 270-388, Wiley, New York.

Kim C.-J., Nelson C. R. (1999), *State-space models with regime switching: classical and Gibbs-sampling approaches with applications*, The MIT Press, Cambridge (MA).

Ladiray D., Mazzi G. L., Sartori F. (2003), *Statistical methods for potential output estimation and cycle extraction*, Working paper and studies, Office for Official Publications of the European Communities, Luxembourg.

OECD (2013), Oecd.Stat, retrieved at http://dx.doi.org/10.1787/data-00285-en on August 31, 2013.

Roeger W. (2006), *The production function approach to calculating potential growth and output gaps estimates for EU member states and the US*, in the workshop on Perspectives on Potential Output and Productivity Growth, Banque de France and Bank of Canada.

Shaikh A. (1974), *Laws of production and laws of algebra: the Humbug production function*, The Review of Economics and Statistics, 5(1), 115-120.

Zellner A., Kmenta J., Dreze J. (1966), *Specification and estimation of Cobb-Douglas production function models*, Econometrica, 34(4), 784-795.

Annex 1: Kalman and Hodrick-Prescott filtering

When dealing with time series, it is necessary to treat raw data, to adjust for seasonalities, to smooth the serie, to de-trend it, etc. This subsection deals with two very common filtering techniques: the Kalman filter and the Hodrick–Prescott (HP) filter. These two techniques are widely used in the potential output computation, we thus retain useful this quick aide-mémoire.

Kalman filter

The Kalman filtering techniques (Kalman, 1960, 1963) and state space representation are usually applied when dealing with latent variables[9]. Each state space representation is composed of a measurement equation which describes the relation between observed variables y_t and β_t, the unobserved state variables (10); and a transition equation describing the dynamics of the state variables (11).

$$y_t = H_t\beta_t + e_t \tag{10}$$

$$\beta_t = \tilde{\mu} + F\beta_{t-1} + v_t \tag{11}$$

Where e_t and v_t are errors, having the following properties:

$$e_t \sim \text{ i.i.d.} N(0, R) \tag{12}$$

$$v_t \sim \text{ i.i.d.} N(0, Q) \tag{13}$$

$$E(e_t v_s') = 0 \tag{14}$$

The Kalman filtering techniques are based on two subsequent phases: a prediction phase followed by an updating one. During the prediction phase, at time t, the unobserved variables $\beta_{t|t-1}$ are predicted using (11), based on previous observation from time 1 to $t-1$. A prediction of the observed variable $y_{t|t-1}$ is then computed based on (10), which allows to determine the prediction error at time t: $\eta_{t|t-1} = y_t - y_{t|t-1}$.

[9] For more information on the Kalman filter and state space representation, see Hamilton (1994) and Kim and Nelson (1999), among others.

This prediction error is used during the updating phase to compute the prediction of the unobserved variable $\beta_{t|t}$, based on observation up to time t. The updating equation is a weighted sum of $\beta_{t|t-1}$ and $\eta_{t|t-1}$ where the weight is given by the Kalman gain K_t. Each new observation enables to update the unobserved variable and from that, to build its time series.

Hodrick-Prescott filter

Hodrick and Prescott (1997) present a technique to obtain the trend of a given serie y_t. Assuming that a time serie y_t may be decomposed in a growth component g_t and a cyclical component c_t (15), they compute g_t such that it minimises (16)[10]

$$y_t = g_t + c_t \tag{15}$$

$$\min_{\{g_t\}_{t=1}^{T}} \left[\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_t - g_{t-1})]^2 \right] \tag{16}$$

where λ is the *Smoothness Parameter* which penalizes variability in the growth component series. Once the state space representation of (15) and (16) is obtained, it is easy to use Kalman filtering techniques to compute the Hodrick-Prescott (HP) trend g_t of the time serie y_t. The HP filter is thus a particular case of Kalman filter where (16) is the transition equation while (15) is the measurement equation.

The HP filter can be used to forecast y_{t+1} in period t, once the trend value in t is given. As the growth component is a second order random walk and because the cycle is not modelled, it follows that the optimal forecast for y_{t+1} is equal to:

$$\hat{g_{t+1}} = 2g_t - g_{t-1} \tag{17}$$

[10] Hodrick and Prescott justify the choice of the minimisation equation as a smoothness measure of gt.

Annex 2: Time series

Table 4: Time series.

Symbol	Name	Description	OECD Table
EMPL	Total Employment	Thousands of persons	ALFS Summary tables
GDP	Gross Domestic Product (expenditure approach)	National currency, current price, millions	Gross Domestic Product
GEXP	Total General Government Expenditure	National currency, current price, millions	Government deficit/surplus, revenue, expenditure and main aggregates
GKF	Gross Capital Formation	National currency, current price, millions	Gross Domestic Product
GREV	Total General Government Revenue	National currency, current price, millions	Government deficit/surplus, revenue, expenditure and main aggregates
GrGDP	Gross Domestic Product growth	growth rate of GDP	Gross Domestic Product
HOURS	Average annual hours actually worked per worker	Number of hours worked per year per person in employment.	Average annual hours actually worked per worker
LF	Total Labour Force	Thousands of persons	ALFS Summary tables
POP	Population	Thousands of persons	ALFS Summary tables
PROD	Labour productivity per person employed	Real output (gross value added) divided by total employed persons	Unit labor costs – annual indicators
UNEMPL	Unemployment	Thousands of persons	ALFS Summary tables
WAGE	Average annual wage	Average annual wages per full-time and full-year equivalent employee in the total economy	Average annual wages
WS	Labour income share (Real ULC)	Total labour costs divided by nominal output	Unit labor costs – annual indicator

Source: OECD symbols and descriptions (2012).

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PRODUCTION FUNCTIONS BEHAVING BADLY RECONSIDERING FISHER AND SHAIKH

Abstract

Here we reconsider Anwar Shaikh's critique of the neoclassical theory of growth and distribution based on its use of aggregate production functions. This is done by reconstructing and extending Franklin M. Fisher's 1971 computer simulations, which Shaikh used to support his critique. Together with other recent extensions to Shaikh's seminal work, the results support and strengthen the evidence against the use of neoclassical *aggregate production functions.*

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1. Introduction

The notion of aggregate production functions has long been a widely used theoretical concept in economics and remains among the fundamental concepts presented in almost every course in micro and macroeconomics. Furthermore, aggregate production functions constitute the core of the supply side in most modern econometric as well as theoretical models, e.g., *CGE* models.

In 1974, Anwar Shaikh proposed a serious critique of the neoclassical theory of growth and distribution based on its use of aggregate production functions. Empirical studies had hitherto shown that aggregate production functions of the Cobb-Douglas type usually fit the data well, and that the estimated coefficients typically coincide with observed wage and profit shares of income. These empirical findings were used not only to support the neoclassical theory of growth and distribution, but also to contest non-micro founded theory, because of its lack of this kind of "indisputable" support.

However, as Shaikh (1986, p. 191) claims the "apparent empirical strength of aggregate production functions is often interpreted as support for neoclassical theory. But there is neither theoretical nor empirical basis for this conclusion."

The purpose of this chapter is to reconsider Shaikh's critique (Shaikh, 1974; see also Shaikh, 1980, 1986, 2005) and parts of the subsequent work on the subject. This is done by reconstructing and extending the original computer simulations by Fisher (1971), which Shaikh used to support his thesis. We extend Fisher's simulations by introducing CES production functions at the industry level, but continue to estimate a simple Cobb-Douglas production function from the aggregated data. This, we claim, provides further insight into the extent and implications of Shaikh's critique. We will show that Fisher's simulation experiment can be reconstructed and by doing this we can also confirm Fisher's 1971 findings, which in itself is of interest because these results have been widely used, but to the best of our knowledge never verified. Furthermore, by inspecting the goodness of fit in Fisher's almost 1000 experiments, we will show that Shaikh's interpretation of Fisher's work is correct. Finally, we compare our results with those obtained by McCombie and Dixon (1991), Felipe and Holz (2001), and Shaikh (2005). In line with these researchers, we find evidence to support a more general version of Shaikh's original critique.

Section 2 and 3 present Shaikh's original critique and subsequent extensions, Section 4 and 5 deals with Fisher's original model and the reconstruction, and Section 6 presents the extension of Fisher's model. Section 7 concludes the paper with a discussion on the consequences of this critique for the neoclassical theory of growth and distribution.

2. Laws of algebra

Shaikh claims and proves that whenever input–output data exhibit constant income shares, there is a very good chance that regardless of the true nature of the data, an aggregate production function of the Cobb-Douglas type will fit the data very well. Therefore, Shaikh concludes that when one estimates a Cobb-Douglas production function on input–output data, there is a good chance that one only observes *laws of algebra* and not *laws of production*.

The following is a concise version of Shaikh's proof. It starts with the universal income accounting identity, *viz*.

$$Y = wL + rK \tag{2.1}$$

Let $y = Y/L$, $k = K/L$, $\alpha = rK/Y$, $1 - \alpha = wL/Y$, and assumes that labour's share of income is constant over time. Now (2.1) can be written as $y = w + rk$.

$$\begin{aligned} y = w + rk \Rightarrow \dot{y} = \dot{w} + \dot{r}k + r\dot{k} \Leftrightarrow \frac{\dot{y}}{y} &= \frac{w}{y}\frac{\dot{w}}{w} + \frac{rk}{y}\frac{\dot{r}}{r} + \frac{rk}{y}\frac{\dot{k}}{k} \\ \Rightarrow \frac{\dot{y}}{y} &= (1-\alpha)\frac{\dot{w}}{w} + \alpha\frac{\dot{r}}{r} + \alpha\frac{\dot{k}}{k} \\ \Rightarrow \ln y &= (1-\alpha)\ln w + \alpha \ln r + \alpha \ln k + \ln c_0 \\ \Rightarrow y &= C_1 k^\alpha \Leftrightarrow Y = C_1 K^\alpha L^{1-\alpha} \end{aligned} \tag{2.2}$$

Where the shift term C_1 is given by:

$$C_1 = c_0 \cdot \ r^{\alpha} w^{1-\alpha} \tag{2.3}$$

To sum up, from a tautology of input–output data and an assumption of constant input shares (plus an implicit assumption of differentiable functions), a function of the Cobb-Douglas type follows directly through basic applications of the laws of algebra! This is an important result, since it implies that regressions of a Cobb-Douglas production function, given that the data exhibit constant input shares, are predetermined to give high correlation coefficients, and are thereby meaningless.

Because of this Shaikh named the Cobb-Douglas production function the "HUMBUG" production function, and emphasized the message by showing that the coordinates in the Cartesian plane spelling the word "HUMBUG" together with profit shares from the US (Solow's 1957 data) could be fitted almost perfectly by a Cobb-Douglas production function (Shaikh, 1974).[1]

3. Related work

The use of aggregate production functions has long been a subject of serious discussion, and no consensus has yet been reached. The debate can be divided into two major parts: the so-called *index number problem* and *value problem*, which respectively refer to the problems of aggregation and the logical problem in determining the value of capital independently of the profit rate. Here we deal only with the index number problem, or to be more specific, the issues of interpreting aggregated empirical results from technologically diverse economies.[2]

Following the first paper by Shaikh on the HUMBUG production function, a number of theoretical and empirical studies have been

[1] Shaikh's results have been challenged by Solow (1974), but subsequently defended by Shaikh (1980), after which the discussion, to the best of our knowledge, seems to have gone quiet.

[2] See Cohen and Harcourt (2003) for a extensive survey and Zambelli (2004) for a more concise survey on the value problem.

published on the subject with J. Felipe and J.S.L. McCombie as the main contributors (see Felipe and Fisher, 2003).

However, the papers by McCombie and Dixon (1991), Felipe and Holz (2001), and Shaikh (2005) are of special interest for this article. McCombie and Dixon (1991) prove that Shaikh's critique also stands when factor shares are not constant as long as the shift term grows with a constant rate. Furthermore, they show that even if the shift term does not grow with a constant rate, it is possible 'with sufficient ingenuity, to find a functional form which will produce a very good fit to the underlying identity' McCombie and Dixon (1991, p. 40), and they refer to the CES and the translog production function as potential candidates.[3]

The paper by Felipe and Holz (2001) presents an interesting Monte Carlo simulation that shows *"that the Cobb-Douglas form is robust to relatively large variations in the factor shares. However, what makes this form quite often fail are the variations in the growth rates of the wage and profit rates. The weighted average of these two growth rates has been shown to be the coefficient of the time trend. This implies that, in most applied work, a Cobb-Douglas form (i.e. approximation to the income accounting identity) should work. We just have to find which Cobb-Douglas form with a dose of patience in front of the computer."* (Felipe and Holz, 2001, p. 281).

Moreover, they show that spurious regression cannot explain the systematic (near) perfect fit of the Cobb-Douglas function.

In a recent paper, Shaikh (2005) presents a more general version of his original results; the so-called *Perfect Fit Theorem*. This theorem states that, given a stable labour share, it is always possible to construct a time function $F(t)$, 'that will always make fitted production functions work "perfectly" in the sense of Solow: that is, make them yield perfect econometric fits with partial derivatives that closely approximate observed factor prices' Shaikh (2005, p. 457). The time function must merely be constructed in the following way:

[3] See also Felipe and McCombie (2001) for a very interesting study of the CES production function's ability to fit input–output data, where they reconsider Arrow *et. al.* (1961) seminal work on the CES function.

$$SR_t = \alpha_{t-1}\Delta \log r + (1-\alpha_{t-1})\Delta \log w \tag{3.1}$$

$$F_t = \beta + h\left(SR_t - \tfrac{1}{t}\sum SR_t\right) \tag{3.2}$$

Note that (3.1) resembles the shift term (2.3) and that an affine function of the Solow Residual SR_t yields an affine time function F_t.

Coherently with McCombie and Dixon (1991) and Felipe and Holz (2001), we agree that the assumption of constant input shares is not needed for the main results to hold. However, we think that the Cobb-Douglas production function fits the data well even when shares are not stable and the shift term (2.3) fails in a test of trend stationarity.

As for Shaikh's Perfect Fit Theorem, we acknowledge the power of the theorem in its ability to ensure a perfect fit, but we also underline that constant input–shares are still a required assumption.

It is important to note that our results are not conditioned on a perfect fit, but on a good fit, by which we mean a fit that would make most econometricians, given the usual reservations, accept the model as a good description of the data. In other words, we are not per se interested in the theoretical — but very possible — possibility of making a neoclassical production function fit the data perfectly, with the help from cleverly constructed trend terms or more flexible functional forms such as the CES or translog. We are merely interested in the basic method of regressing a log-linearized Cobb-Douglas function with a simple (affine) trend term on input– output data, and will show that this method often is sufficient to ensure a good fit, even when the underlying data should not be explainable with such a simple model. We believe this is an interesting approach, because this method is extensively used by not only students of economics, but also established researchers. Showing that these claims hold will be the main quest in the following.

4. Fisher's model

The purpose of Fisher's 1971 paper was to study the conditions under which the production possibilities of a technologically diverse economy can be represented by an aggregate production function.

The work consists of a huge simulation experiment, where production is simulated at the micro level in a neoclassical model with n heterogeneous firms — all possessing Cobb-Douglas technology. Labour is assumed to be perfectly mobile, but capital and technology are bound to the respective firms. Wage and profit are as usual given by the marginal productivity of labour and capital, respectively. Furthermore, it is assumed that through perfect competition the labour inputs in each period are distributed such that wages would be uniform.

The experiments are divided into two major groups: the so-called *Capital experiments* in which economic development is based on the evolution in the stock of capital, and the *Hicks experiments* in which development is based on changes in a Hicks neutral technology. The experiments were divided into a total of five subgroups depending on the underlying pattern of technological progress. The experiments ran over 20 periods with two, four, or eight firms, and for each experiment three different initial capital or technology endowments, two choices of weights in the production function, and eleven different growth rates in capital or technology were chosen. This gives a total of 990 (5 × 3 × 3 × 2 × 11) unique experiments. See Appendix A for further details.

The experiments were constructed in order to systematically violate the conditions for a theoretically consistent aggregation; see Fisher (1969) for a discussion of these conditions. Capital is aggregated using the profit rates, *viz*.

$$J_t = \sum_{i=1}^{n} \left(\frac{\sum_{t=1}^{20} r_{i,t} K_{i,t}}{\sum_{t=1}^{20} K_{i,t}} \right) K_{i,t} \quad i = 1, 2, ..., n \quad t = 1, 2, ..., 20 \tag{4.1}$$

The aggregate Cobb-Douglas production function is given by

$$Y_t = A_t J_t^{\alpha} L_t^{1-\alpha} \tag{4.2}$$

w_t

4.1 Evaluation of the model

The primary measurement of performance is the relative root-mean-square error together with the standard deviation of labour's share, *viz*.

$$S = \frac{\sqrt{\frac{1}{20}\sum_{t=1}^{20}(w_t - \hat{w}_t)^2}}{\frac{1}{20}\sum_{t=1}^{20} w_t} \tag{4.3}$$

$$\sigma_\alpha = \sqrt{\frac{1}{20-1}\sum_{t=1}^{20}\left(\hat{\alpha}_t - \frac{1}{20}\sum_{t=1}^{20}\hat{\alpha}_t\right)^2} \tag{4.4}$$

Where $\hat{\alpha}$ and $\hat{w}$ denotes estimated values.

In relation to the analysis of Shaikh's thesis, the standard deviation of labour's share $\sigma\alpha$ is important, because of the assumption of a constant labour share.

The parameter α in the aggregate production function is estimated from the following simple log-linearized model:

$$\ln \frac{Y_t}{L_t} = \beta_1 + \beta_2 t + \alpha \ln \frac{J_t}{L_t} + \epsilon_t \tag{4.5}$$

The work presented in Section 3 would predict that the correlation coefficients from the above regression will be equal to or very close to one, whenever the input– output data exhibits either (A) constant factor shares or (B) factor shares that change so that the shift term, see equation (2.3), grows at a constant rate. It is these conditions, we attempt to investigate below.

The trend term $\beta_2 t$ is included to capture what can be characterised as a constant growth in the (aggregated) Hicks neutral technology. Following Fisher (1971, p. 313) this trend term is only included in the Hicks experiments.

To check whether or not assumption A and/or B are satisfied in the experiments, the following methods are used. Constant factor shares are checked by the standard deviation of labour's share $\sigma\alpha$; if this is sufficiently small, it would seem reasonable to accept assumption A. As for assumption B, equation (2.3) states that the shift term is given by a weighted average of the wage and the profit rate; i.e., testing assumption B is equivalent to testing whether or not the following variable is a trend stationary time series.

$$C_t = r_t^{\alpha_t} w_t^{1-\alpha_t} \tag{4.6}$$

Where $1 - \alpha_t = \frac{L_t w_t}{Y_t}$ and $r_t = \frac{\sum r_{i,t}}{J_t}$. As usual this is done by including a trend term in the ADF test. Details will be given in the following sections.

5. The reconstruction

It cannot be expected that the reconstruction yields a perfect replication of Fisher's work, because we do not have information on the pseudo-random number generating algorithm[4].

For reasons of comparability, the original and the reconstructed data are presented in the same type of matrices as Fisher used. These matrices sum up the frequency of observations with a given combination of σ_α and S.

There are some deviations, but these deviations can be justified by the stochastic elements in the model. In any case, Fisher's basic observation is confirmed, i.e., an aggregate production function often provides a good explanation of wages, provided that the input weights are relatively stable over time. Given Fisher's earlier work on the subject (Fisher, 1969), these results must have been surprising, as the following quote also suggests: *The point of our results, however,*

[4] It is fairly easy to describe the simulations, because The simulation are based on the thorough documentation present in Fisher (1971). We have used MATLAB to write three small programs, which are available upon request.

These programs consist of a master m-file, which basically is Fisher's model as described in his paper plus the extensions. These programs also contain algorithms performing different methods for evaluation, e.g., a set of loops that automatically perform standard ADF tests for stationarity by calculating test statistics and comparing these with the appropriate table values. The significance level for all tests is 5 percent.

The set-up of the experiments is programmed in another m-file, e.g., the different combinations of exogenously given parameter values. Furthermore, this program collects and organises the output. The last m-file is a wage-equilibrating-algorithm, which is used because in every period in every experiment the wage rates must be uniform among the *n* firms; see Fisher (1971, p. 308) or Appendix A for further details. The wage-equilibrating-algorithm is extremely time-consuming due to inefficient programming and computational complexity.

Table 1: Summary of the Capital and Hicks experiments.

S/σ_α	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	> 3.0
0.0-0.5	296	1	1	0	0	0	0
0.5-1.0	104	91	2	0	0	0	0
1.0-1.5	26	31	41	3	0	0	0
1.5-2.0	13	7	26	20	5	0	0
2.0-3.0	13	11	12	18	19	14	5
3.0-4.0	6	11	4	6	5	2	16
4.0-5.0	3	2	6	3	2	0	13
5.0-10.0	5	14	6	9	7	8	23
10.0-20.0	0	5	6	5	0	4	13
>20.0	1	1	8	5	2	3	26

Source: original data.

Table 2: Summary of the Capital and Hicks experiments.

S/σ_α	0.0–0.5	0.5–1.0	1.0–1.5	1.5–2.0	2.0–2.5	2.5–3.0	> 3.0
0.0–0.5	290	0	0	0	0	0	0
0.5–1.0	122	40	0	0	0	0	0
1.0–1.5	32	50	14	0	0	0	0
1.5–2.0	7	21	37	4	0	0	0
2.0–3.0	24	16	10	17	14	0	0
3.0–4.0	5	10	7	3	6	11	1
4.0–5.0	5	6	12	2	1	2	9
5.0–10.0	14	22	12	9	7	5	15
10.0–20.0	2	11	7	6	8	8	8
>20.0	1	4	10	6	5	7	46

Source: reconstructed data.

is not that an aggregate Cobb-Douglas fails to work well when labor's share ceases to be roughly constant, it is that an aggregate Cobb-Douglas will continue to work well so long as labor's share continues to be roughly constant [...] (Fisher 1971, p. 307).

This reconstruction of Fisher's work allows us to examine the goodness of fit of the underlying regressions. Note that confirming Fisher's original results is *per se* useful, since several authors over the years have referred to these results.

Inspecting the correlation coefficients from Equation 4.5 and standard deviations of labour's share from the 990 unique experiments show that almost all correlation coefficients are very close to 1; 98 percent are greater than 0.90 and 85 percent are greater than 0.99. Moreover, the correlation does not seem to decrease as σ_α increases. Even more interesting, the 96 series with non-trend stationary shift terms continue to give high correlation coefficients: 95 percent of all correlation coefficients are greater than 0.90 and 78 percent are greater than 0.99.

These observations imply that Shaikh's *law of algebra* may very well be more general than formally constrained by assumption A, constant labour shares, and assumption B, a constant growth rate in the shift term.

Note however, that (near) perfect correlation is not always observed, but $R^2 > 0.90$ would lead most researchers, given the usual reservations, to (in this case wrongly) conclude that the estimated model is a good explanation of the underlying system.

To avoid any misconceptions, these results do not contradict those of Shaikh or the subsequent work presented in Section 3, they show that in applied work the risks of making wrong conclusions are not restricted to the cases where assumptions A and B are satisfied.

To ensure that these high correlations are not observations of spurious regressions, the explanatory and the dependent variables in equation (4.5) are checked for possible unit roots by a simple ADF tests. From this it is inferred whether or not there is a potential risk for spurious regression, i.e., if both the dependent and explanatory variables have a unit root. These tests shows that there is only a potential risk for spurious regression in 5.2 percent of the 990 regressions, i.e., the high correlation coefficients cannot be explained by spurious regression. This result is consistent with Felipe and Holz (2001), who also conclude that spurious regression cannot explain the uniformly high fit.

6. The extended model

In the following, an extension of Fisher's model is employed to further investigate the generality of Shaikh's critique. The model is changed by replacing the micro Cobb-Douglas production functions with CES production functions, but still estimating an aggregate Cobb-Douglas production function after the conditions for a theoretically consistent aggregation are violated as in the original model.[5] The CES production function is of the following form, where ν is the reciprocal of the elasticity of substitution between capital and labour σ_{KL}, *viz*

$$y_{i,t} = A_{i,t}\left(\alpha_i K_{i,t}^{1-\nu} + (1-\alpha_i)L_{i,t}^{1-\nu}\right)^{\frac{1}{1-\nu}} \tag{6.1}$$

The elasticity of substitution is chosen to be 0.20, 0.40, 0.60 or 0.80. The experiments are in every other way identical to Fisher's, i.e., a total of 3960 (4 × 990) unique experiments.

However, a minor problem emerges: in 760 experiments it was not possible to ensure uniform wages in every period through the redistribution of labour between the n firms. This is a consequence of an obvious mathematical property of the CES function, when capital and technology are ex ante given. To circumvent this problem, all of these experimental sessions, in which it was not possible to determine a set of uniform wage rates in one or more periods, have been removed. Consequently, the following results are based on 3200 (3960 − 760) experiments.

Inspecting the correlation coefficients and standard deviations of labour's share from these 3200 unique experiments show that 81

[5] Fisher et. al. (1977) analysed wage explanation in simulations with CES micro production functions. In this study aggregate Cobb-Douglas as well as CES production functions were estimated and in general both types fit the data well, as long as the shares were stable.

percent of the correlation coefficients are greater than 0.90 and 59 percent are greater than 0.99. In the 595 time series with a non-trend stationary shift term, 80 percent are greater than 0.90 and 44 percent are greater than 0.99. Moreover, there is no clear connection between the standard deviation of labour's share and the correlation coefficients, i.e., again it is shown that under very general circumstances, there is a high risk that this kind of empirical work will result in fundamentally misleading conclusions about the underlying technology.

That *only* 44 percent of the series with a non-trend stationary shift are greater than 0.99 emphasises; that these findings do not generalise Shaikh's result that guaranties a perfect fit under the more restrictive conditions, but simply imply that it is very likely to obtain a very good fit under very general circumstances.

Again the series are checked for potential spurious regressions. The tests show that there is potential risk of spurious regression in 7 percent of the 3200 regressions, i.e., the high correlation coefficients again cannot be explained by spurious regression.

To sum up, the results from the extended model also support a more general version of Shaikh's critique, because even though the likelihood of observing near perfect correlation drops, when assumptions A and B are violated, it is still very likely to obtain correlation coefficients that most researchers would (wrongly) interpret as support for the estimated functional form.

7. Concluding remarks

Fisher's 1971 computer experiment has been reconstructed and his results verified. Strengthened by the extensions we have employed, Shaikh's original findings have been confirmed along with the extensions presented in Dixon and McCombie (1991) and Felipe and Holz (2001). We show that even under the general circumstances where neither Shaikh's Perfect Fit Theorem nor the results presented in Dixon and McCombie (1991) and Felipe and Holz (2001) would predict a (near) perfect fit, the Cobb-Douglas production function still shows an "impressive" ability to mimic the data, even with the most simple and popular econometric method.

The implications of these cumulative results are important because they imply that empirical studies, in which a Cobb-Douglas production function is estimated, are necessarily inconclusive. This undermines empirical support for the neoclassical theory of growth and distribution, because that support — to a wide extent — is based on the Cobb-Douglas production function. Moreover, it is a serious warning against using *AS IF* justifications for economic theory.

The lesson from this exercise should be that extreme caution is necessary when applying aggregate production functions; indeed, instead of aggregate production functions, we would propose the implementation of (physical) multi-sector input– output systems in general macroeconomic models, because there is neither theoretical nor empirical support for the use of aggregate production functions. In our opinion, an aggregate production function is simply a notion used for mathematical convenience and elegance.

Some might argue that a more "realistic" production function like the (nested) CES or translog would overcome these problems, but the Cobb-Douglas function's ability to fit (plausible and implausible) data are of course fully embedded in the more flexible functional forms.

References

Arrow K. J., Chenery H. B., Minhas B. S., Solow R. M. (1961), *Capital-labour substitution and economic efficiency*, The Review of Economics and Statistics, 43(3), 225-250.

Cohen A. J., Harcourt G. C. (2003), *Retrospectives: whatever happened to the Cambridge capital theory controversies?*, The Journal of Economic Perspectives, 17(1), 199-214.

Felipe J., Fisher F. M. (2003), *Aggregation in production functions: what applied economists should know*, Metroeconomica, 54(2), 208-262.

Felipe J., Holz C. A. (2001), *Why do aggregate production functions works? Fisher's simulations, Shaikh's identity and some new results*, International Review of Applied Economics, 15(3), 261-285.

Felipe J., McCombie J. S. L. (2001), *The CES production function, the accounting identity, and Occam's razor*, Applied Economics, 33(10), 1221-1232.

Felipe J., McCombie J. S. L. (2007), *On the rental price of capital and the profit rate: the perils and pitfalls of total factor productivity growth*, Review of Political Economy, 19(3), 317-345.

Fisher F. M. (1969), *The existence of aggregate production functions*, Econometrica, 37(4), 553-577.

Fisher F. M. (1971), *Aggregate production functions and the explanation of wages: a simulation experiment*, The Review of Economics and Statistics, 53(4), 305-325.

Fisher F. M., Solow R. M., Kearl J. M. (1977), *Aggregate production functions: some CES experiment*, The Review of Economics Studies, 44(2), 305-320.

McCombie J. S. L., Dixon R. (1991), *Estimating technical change in aggregate production functions: a critique*, International Review of Applied Economics, 5(1), 24-46.

Shaikh A. (1974), *Laws of production and laws of algebra: the Humbug production function*, The Review of Economics and Statistics, 56(1), 115-120.

Shaikh A. (1980), *Laws of production and laws of algebra: the Humbug II*, in Nell E. J. (ed.), Growth, profit, and property: Essays in the Revival of Political Economy, Cambridge: Cambridge University Press, 80-95.

Shaikh A. (1986), *Humbug production function*, in Eatwell J., Milgate M., Newman P. (eds.), *The new Palgrave: a dictionary of economic theory and doctrine*, Macmillan, London.

Shaikh A. (2005), *Nonlinear dynamics and pseudo-production functions*, Eastern Economic Journal, 31(3), 447-466.

Solow R. M. (1957), *Technical change and the aggregate production function*, The Review of Economics and Statistics, 39(3), 312-320.

Solow R. M. (1974), *Laws of production and laws of algebra: the Humbug production function: a comment*, The Review of Economics and Statistics, 56(1), 121.

Zambelli S. (2004), *The 40% neoclassical aggregate theory of production*, Cambridge Journal of Economics, 28(1), 99-120.

Annex 1: the experiments

Fisher (1971) presents a neoclassical model comprising of *n* firms each producing one homogeneous output. The inputs consist of homogeneous and perfectly mobile labour and heterogeneous capital and technology that are bound to the individual firms. The experiments run for 20 periods, $t = 1, 2, \ldots, 20$.

Production at the *i*th firm is either modelled by a Cobb-Douglas or CES production function, *viz.*

$$y_{i,t} = A_{i,t} L_{i,t}^{\alpha_i} K_{i,t}^{1-\alpha_i} \tag{A.1}$$

$$y_{i,t} = A_{i,t} \left[\alpha_i L_{i,t}^{1-\nu} + (1-\alpha_i) K_{i,t}^{1-\nu} \right]^{\frac{1}{1-\nu}} \tag{A.2}$$

Where v is the reciprocal of the elasticity of substitution between labour and capital. The aggregate production function and the associated aggregate capital stock is given by:

$$Y_t = A_t L_t^{\alpha} J_t^{1-\alpha} \tag{A.3}$$

$$J_t = \sum_{i=1}^{n} \left(\frac{\sum_{t=1}^{20} r_{i,t} K_{i,t}}{\sum_{t=1}^{20} K_{i,t}} \right) K_{i,t} \tag{A.4}$$

Wages and profits are paid their marginal products and it is assumed that labour is distributed such that the wage level coincides across the *n* firms. The algorithm applied to ensure distribution of labour is presented.

In all experiments, the evolution of the total supply of labour, Hicks neutral technology, and capital endowments are exogenously given by:

$$L_t = \exp(0.3t + 0.02\varepsilon_t) \qquad \varepsilon_t \sim \mathrm{N}(0,1) \tag{A.5}$$

$$A_{i,t} = \exp(\gamma_{i,1} t) \qquad v_{i,t} \sim \mathrm{N}(0,1) \tag{A.6}$$

$$K_{i,t} = \exp(\beta_{i,0} + \beta_{i,1} t + 0.0001\eta_{i,t}) \qquad \eta_{i,t} \sim \mathrm{N}(0,1) \tag{A.7}$$

The experiments include two, four, or eight firms. Depending on this, the parameter, α_i, from the production function can take the following values:

$$n = 2: \quad (\alpha_1, \alpha_2) \in \{(0.7, 0.8), (0.6, 0.9)\}$$

$$n = 4: \quad (\alpha_1, ..., \alpha_4) \in \{(0.6, 0.7, ..., 0.9), (0.7, 0.725, ..., 0.8)\}$$

$$n = 8: \quad (\alpha_1, ..., \alpha_8) \in \{(0.6, 0.6 + \tfrac{1}{7}0.3, ..., 0.9), (0.7, 0.7 + \tfrac{1}{7}0.1, ..., 0.8)\}$$

The initial capital endowments can be distributed in three different ways, *viz.*

1. $$\beta_{i,0} = 0 \;\; \forall \;\; i = 1, 2, ..., n$$

2. $$\begin{aligned} \beta_{i,0} &= 0 \;\; \forall \;\; i = 1, 2, ..., \tfrac{n}{2} \\ \beta_{i,0} &= 2 \;\; \forall \;\; i = \tfrac{n}{2}, \tfrac{n}{2} + 1, ..., n \end{aligned}$$

3. $$\begin{aligned} \beta_{i,0} &= 2 \;\; \forall \;\; i = 1, 2, ..., \tfrac{n}{2} \\ \beta_{i,0} &= 0 \;\; \forall \;\; i = \tfrac{n}{2}, \tfrac{n}{2} + 1, ..., n \end{aligned}$$

Finally, the experiments fall into the following five groups:

1. Two group capital

$$\begin{aligned} \beta_{i,1} \in \{-0.05, -0.04, ..., 0.05\} &\quad \forall \;\; i = 1, 2, ..., \tfrac{n}{2} \\ \beta_{i,1} = 0 &\quad \forall \;\; i = \tfrac{n}{2}, \tfrac{n}{2} + 1, ..., n \\ \gamma_{i,1} = 0 &\quad \forall \;\; i = 1, 2, ..., n \end{aligned}$$

2. Two group Hicks preliminary

$$\begin{aligned} \gamma_{i,1} \in \{-0.05, -0.04, ..., 0.05\} &\quad \forall \;\; i = 1, 2, ..., \tfrac{n}{2} \\ \gamma_{i,1} = 0 &\quad \forall \;\; i = \tfrac{n}{2}, \tfrac{n}{2} + 1, ..., n \\ \beta_{i,1} = 0 &\quad \forall \;\; i = 1, 2, ..., n \end{aligned}$$

3. Two group Hicks

$$\begin{aligned}
\gamma_{i,1} &\in \{(\alpha_i - 1)0.05, (\alpha_i - 1)0.04, ..., (1 - \alpha_i)0.05\} && \forall \ \ i = 1, 2, ..., \tfrac{n}{2} \\
\gamma_{i,1} &= 0 && \forall \ \ i = \tfrac{n}{2}, \tfrac{n}{2} + 1, ..., n \\
\beta_{i,1} &= 0 && \forall \ \ i = 1, 2, ..., n
\end{aligned}$$

4. The fanning capital

$$\begin{aligned}
\beta_{i,1} &\in \{(i - 1)0.05, (i - 1)0.04, ..., (i - 1)0.05\} && \forall \ \ i = 1, 2, ..., n \\
\gamma_{i,1} &= 0 && \forall \ \ i = 1, 2, ..., n
\end{aligned}$$

5. The fanning Hicks

$$\begin{aligned}
\gamma_{i,1} &\in \{(\alpha_i - 1)(i - 1)0.05, (\alpha_i - 1)(i - 1)0.04, ...,(1 - \alpha_i)(i - 1)0.05\} \\
& \qquad \forall \ \ i = 1, 2, ..., n \\
\beta_{i,1} &= 0 \qquad \forall \ \ i = 1, 2, ..., n
\end{aligned}$$

This concludes the description of the experiments which by simple combinatorial calculation amounts to 990 unique settings.

The algorithm, used to distribute labour among the *n* firms such that the wage levels are approximately equal across the firms, has the following structure.

1. Distribute the initial endowments of capital and technology.
2. Uniformly distribute the total labour supply across the *n* firms and compute the *n* wage levels.
3. Allocate a given amount of labour from the firms with a low wage level to the the firms with high wage level.
4. Repeat step three until the maximum deviation among the wage levels are less than 1 percent.

Computation of this, however, is not always straightforward because the production functions satisfy the *Inada Conditions*, i.e., when labour inputs are close to zero small changes have large effect on the marginal products. The solution is to dynamically reduce the allocation of labour as the wage levels converge.

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TECHNOLOGICAL PROGRESS AND THE RATE OF PROFIT

Abstract

This article pays homage to Anwar Shaikh from a perspective which is also treasured by the honoree: a Marxian point of view. The purpose is to set up a simple stylized universe consisting of two goods (from which we can try to generalize about the case of n goods, though I do not do so in this article), and examine in the context of this universe, for which a set of equilibrium prices is determined, how technological progress impacts on the rate of profit. The entire discussion here focuses on equilibria with no demand or labour supply constraints.

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Keywords: Labour productivity, Capital Goods Sector, Consumption Goods Sector, Savings, Profits, Theorem.

1. Itroduction

The pervasive assumption in neo-classical growth theory is of Harrod-neutral technological progress, which is defined as a state where, for any given rate of profit, technological progress leaves the capital-output ratio unchanged. Since neo-classical economics assumes a production function, along which the rate of profit, being equal to

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the marginal product of "capital", varies from one point to the next, Harrod-neutral technological progress refers to a shift in the production function, such that this condition is satisfied for every point.

Of course there never is a production function of the neo-classical kind, with an infinite number of techniques. It can at best be seen as a stylized presentation of a world where there are a few discrete techniques and the concept of marginal productivity has little meaning. But even the idea of discrete techniques across which the economy can move in *logical time,* makes little sense. It is best therefore to visualize the economy as being at some particular technique (with fixed coefficients), and technological progress as occurring through historical time to reduce these coefficients per unit of output.

Defining Harrod neutrality in the usual way in such a world makes little sense, since the rate of profit here is no longer determined by full employment marginal productivity, but in some other way (typically by the bargaining strength of the workers that fixes the real wage). It is more appropriate to define Harrod-neutral technological progress in this economy as a situation where if the wage-share in output remains the same after technological progress as before, then the rate of profit also remains the same. (In fact if we leave aside the Cobb-Douglas case, then this definition of Harrod-neutrality can also be validly used in the neo-classical production function case, since every capital-output ratio along the production function is associated with a unique wage-share except when it is Cobb-Douglas).

Marx did not talk in terms of a production function. Indeed the Marxian discussion of technological progress is also best thought of as a temporal shift from one technique to another. Marx's perception however, in contrast to Harrod-neutrality, was that technological progress was always such as to leave the rate of profit *lower* for any given share of wages in output. *Both Marx's as well as the neo-classical discussion in this context, it should be noted, assumes the absence of any demand constraint.*

To say in the fixed coefficient (one technique) case that technological progress keeps the rate of profit unchanged for a given wage share is the same as saying that the increase in labour productivity through technological progress (with which the real wage rises *pari passu* to keep the wage share unchanged) is associated with an unchanged capital-output ratio. This is essentially what Harrod-neutrality amounts to in a one-good world with fixed coefficients, while Marx's presumption is that technological progress raises labour productivity, but at the cost of a rise in the capital-output ratio. The question is: which of these is a more plausible postulate? Given Anwar Sheikh's long interest in the Marxian proposition of the falling tendency of the rate of profit, I think this is a question that would interest him, which is why I take it up in the present paper written in his honour.

2. A simple stylized universe

There is however an obvious hurdle to be crossed before this question can be discussed. And that consists in the fact that we do not live in a one-good world. The world consists of different commodities, and we cannot discuss the nature and implications of technological progress without bringing in the price-system. The purpose of this paper is to set up a simple stylized universe consisting of two goods (from which we can try to generalize about the case of n goods, though I do not do so in this paper), and examine in the context of this universe, for which a set of equilibrium prices is determined, how technological progress impacts on the rate of profit. The entire discussion here focuses on equilibria with no demand or labour supply constraints.

In any economy where machines are used there must be a sector (or a group of inter-connected sectors) which use machines to produce the same machines. With this idea in mind we divide the economy into two sectors: sector 1 uses a mother machine to produce the same mother machine; sector 2 uses the mother machine to produce a consumer good. Each sector uses only machine and labour, but, for simplicity, no current material inputs. Machines, again for simplicity, are supposed to last for ever. We can set out the production conditions of the two sectors as follows:

Sector 1: Machines + labour produce machines

Sector 2: Machines + labour produce consumer goods

The price equations of the system are as follows:

$$a.p_1 (1+r) + w.l_1 = a.p_1 + p_1 \qquad \text{(i)}$$

$$b.p_1 (1+r) + wl_2 = b.p_1 + p_2 \qquad \text{(ii)}$$

where wages are assumed to be paid at the end of the period, the capital inputs per unit of output (which are also assumed to be jointly produced at the end of the period, with unchanged value owing to the infinite life of capital) are denoted by a and b respectively, the profit and the wage rate have the usual notations, and the labour input per unit of output in each sector is denoted by l with an appropriate subscript.

The price vector in such a system is $p' = w.l' (I - r.B)^{-1}$ where l' is the row vector of labour coefficients and B is the matrix of capital input coefficients (only 2×2 in the present instance). For a given level of w (in terms of either commodity taken as numeraire), there is a unique rate of profit satisfying (i)-(ii) with which strictly positive prices are associated, provided an obvious viability condition is satisfied, namely that the value of the wage bill must not exceed the total value added in the system. To see this let us take the first commodity as the numeraire and let w' be the magnitude of wage in terms of it. Then from (i) it follows that

$$r = (1 - w'.l_1) / a \qquad \text{(iii)}$$

At this r, which is unique there is a unique positive price for the other commodity, namely the consumer good. Now, if the numeraire were changed, with w' still denoting the wage rate in terms of first commodity, then that *per se* would make no difference to the rate of profit or the relative prices. Since this would be true no matter what the level of the wage rate in terms of the first commodity, provided $w'.l_1 \leq 1$, it follows that even if the wage rate were fixed in terms of the other commodity, there would still be a unique rate of profit and a unique set of positive relative prices associated with it, subject to this viability condition. It follows then that for any given wage rate within this bound, (i)-(ii) give a unique profit rate as a meaningful solution.

3. Technological progress

Let us now consider technological progress. Technological progress typically entails a change in the form of the machine, so that in sheer physical terms our sector 1 does not produce the same output *after* technological progress as *before*. Nonetheless we can continue to work with the same notions of capital and labour coefficients as earlier, because the consumer good remains unchanged and the demand for machines may be seen as a derived demand.

The basic effect of technological progress is to increase, for any given rate of profit, the wage rate that can be paid. It does so primarily by increasing labour productivity at the existing r and w, so that the new labour coefficients (denoted by primes) satisfy $l'_1 \leq l_1$, $l'_2 \leq l_2$, *with at least one of these being a strict inequality*. While doing so however it simultaneously alters the capital coefficients. We assume, to start with, that if a' and b' denote the new capital coefficients, then $a' \geq a$, $b' \geq b$, which is a weaker version of Marx's proposition that the essence of technological progress is to substitute "dead labour" for "living labour".

Once labour productivity has increased at the earlier wage rate, the wage rate itself increases in response to the productivity increase. And we assume in common with Marx, as well as Harrod-neutrality, that real wages rise at the same rate as overall labour productivity.

A problem however arises here: when relative prices change because of technological progress, the rates of increase in real wages will differ depending on the commodity in terms of which real wages are being measured; but the rates of increase in labour productivity on the other hand are independent of any such commodity measure. How then do we express the assumption that real wages rise in tandem with labour productivity? In what follows we assume that real wages are expressed in terms of the consumer good and they rise in tandem with the average labour productivity. This average is a weighted average, with the weights corresponding to the relative sizes of the two sectors.

To concretize matters, we assume that the relative sizes prevailing are those that would prevail along (some) steady-growth path, i.e. *that the economy was experiencing steady growth when technological progress occurs*. Let us clarify this point.

Let K_i denote the machine stock in the i-th sector in any period. If the economy is experiencing steady state growth at the rate n, then it must be the case that $dK_i / dt = n.K_i$.

It can be easily seen that in the absence of technological progress K_1, and K_2 must, in steady state, be in the ratio of (a.n) and (1-a.n). Employment in the two sectors on this steady state path will be $K_1.l_1/a$, and $K_2.l_2/b$. Let us denote $(K_1.l_1/a)/ [(K_1.l_1/a) + (K_2.l_2/b)]$ by k, then the ratio of employment will be k and (1-k) respectively in the two sectors. The average labour productivity growth is simply the weighted average of the productivity growths in the two sectors where the weights are k and (1-k) respectively.

More specifically our assumption about the change in wage rate is as follows: denoting by w the wage rate in terms of consumer good, and using primes, as before, for the post technological progress situation, we postulate that

$$w / w' = k.(l_1' / l_1) + (1- k).(l'_2 / l_2) \; \ldots \qquad (T)$$

where l's denote the labour coefficients (the reciprocals of labour productivity). With technological progress of course, not only the labour coefficients but also and the weights k and (1-k) will be changing over time. *But in this paper our concern is with one-shot technological progress.* Hence all these time-dependent variables are mere givens, with clear unambiguous values, when such one-shot technological progress occurs.

We can now establish the following theorem:

Theorem 1: In an economy where technological progress takes the above form, i.e. $l'_i \leq l_i$ with at least one strict inequality, $a \leq a'$, $b \leq b'$, and where the real wage rate in terms of the consumer good increases at the same rate as the weighted average labour productivity, the rate of profit is unaffected by technological progress if (1) $a' = a$, and $b' = b$, and (2) the rate of growth of labour productivity is the same in both the sectors.

Proof: If the growth of labour productivity is the same in both sectors then the real wage rate too will rise at the same rate, which means $w'.l'_1 = w.l_1$, and $w'.l'_2 = w.l_2$. With a'=a and b'=b, this means that technological progress leaves the price equations unchanged, and hence r and p, the price of the consumer good (taking the first good as numeraire).

Q.E.D.

Theorem 1 has dealt with sufficient conditions. We shall also take up the question of necessary and sufficient conditions, but we shall do so by building up to it through a series of steps. Meanwhile a procedural issue needs to be dealt with.

Without any loss of generality we can define a unit of commodity 1 as the output that is produced when a proportion k of the employed work-force is employed in this sector and a unit of commodity 2 as the amount that is produced when a proportion (1-k) of the employed work force is employed in this sector. Now, it is clear from (T) above that the rise in wages as a consequence of technological progress leaves the wage bill in terms of the consumer good unchanged for a given output vector in this economy. Let us denote this wage bill by ε.

Now, there are three possibilities: $\varepsilon = 1$, i.e. the wage bill equals the consumer goods sector output, which entails that savings = profits in the economy, and is equivalent to the classical savings assumption, "all wages are consumed and all profits are saved"; $\varepsilon<1$ when savings < profits, which is equivalent to the assumption "all wages are consumed and a part of profits is saved"; and $\varepsilon > 1$ if savings exceed profits, a less likely scenario which we shall not go into. In what follows, whenever we discuss $\varepsilon < 1$, we assume that all wages and a *fixed proportion of profits* are consumed.

On the basis of this discussion we can advance the following Lemma:

Lemma 1: If a=a', b=b', the rate of profit cannot remain unchanged if labour productivity growth is unequal across sectors, and savings $\neq$ profits.

Proof[1]: Now from the two price equations, which (taking the first commodity as numeraire) can be written as

$$ar + w.l_1.p = 1 \qquad \text{(iv)}$$

and

$$br + w.l_2.p = p, \qquad \text{(v)}$$

it follows that at any given r,

$$p = (1- ar)/ w.l_1 = (1-ar)/ (\varepsilon-w.l_2) = br/(1- w.l_2)$$

where $\varepsilon = \sum w.l_{i.}$ Putting it differently,

$$(1-ar)/br = (\varepsilon- w.l_2)/ (1-wl_2).$$

[1] An alternative proof using continuous time can be put forward as follows:

Proof: If r is to be unchanged, then from the first price equation (first commodity as numeraire) which now can be written as

$$ar + w.l_1.p = 1 \qquad \text{(iv)}$$

it follows that $w.l_1$ p must remain unchanged, or $[d(w.l_1)/dt]/w.l_1 = -[dp/dt]/p$. Since labour productivity growth is unequal across sectors, the l.h.s *ex hypothesi* is not zero, nor therefore is the r.h.s. From the second price equation, which now can be written as

$$br + w.l_2.p = p, \qquad \text{(v)}$$

we get

$$[dp/dt]/p = -[d(1-w.l_2)/dt]/(1-wl_2) = [wl_2/(1-wl_2)][d(wl_2)/dt]/wl_2$$

It follows then that for the profit rate to remain unchanged

$$[d(wl_1)/dt]/wl_1 = - [wl_2/(1-wl_2)]. [d(wl_2)/dt]/wl_2$$

Denoting $wl_2/(1-wl_2)$ at time t by m(t), this means

$$(dw/dt)/w + (dl_1/dt)/l_1 = - m(t)[(dw/dt)/w + (dl_2/dt)/l_2] \qquad \text{(vi)}$$

Recalling that $(dw/dt)/w = - [k.(dl_1/dt)/l_1 + (1-k)(dl_2/dt)/l_2]$, substituting in (vi), and simplifying,

$[(dl_1/dt)/l_1](1-k-km(t)) = [(dl_2/dt)/l_2](1-k-km(t))$, which means either that (1-k-km(t)) = 0 or that the rates of labour productivity growth must be the same in both sectors. If profits are different from savings, i.e. the economy is not on the von-Neumann path, then (1-k-km(t)) is not zero. It follows then that there is no way that r can remain unchanged without labour productivity growth being equal across sectors. Q.E.D.

If r remains unchanged, then the LHS remains unchanged despite technological progress, which means the RHS must also remain unchanged. Now if the RHS remains unchanged then p must remain unchanged and therefore the output composition must remain unchanged, and hence ε must remain unchanged. The condition for r remaining unchanged therefore is simply that

$$(\varepsilon\text{- } w.l_2)/(1\text{-}wl_2) = (\varepsilon\text{-}w'.l_2')\ /(1\text{-}w'.l_2')$$

which is possible, when savings are not equal to profits, i.e. $\varepsilon \neq 1$, only if $w.l_2$ remains unchanged, i.e. $w.l_2 = w'.l'_2$. From (T), this can happen only if the rate of growth of labour productivity is identical across sectors.

Q.E.D.

For the case where savings equal profits, we get the opposite result, as shown below:

Lemma 1': If a=a', b=b', and ε = 1, i.e. savings = profits, then the rate of profit *will* remain unchanged no matter what the pattern of increase of labour productivity across sectors.

Proof[2]: At any given r, we have seen, $(1\text{-}ar)/br = (\varepsilon\text{-}w.l_2)/\ (1\text{-}wl_2)$. If ε = 1, then the RHS is 1, in which case (1-ar) = br.
After technical progress we must have (1-a',r') = b'r'. Since a=a' and b=b', it follows that r = r', i.e. the rate of profit remains unchanged.

Q.E.D.

We thus get two diametrically opposite results in the two cases, one where savings equal profits and the other where they are unequal, on the effect of unequal labour productivity growth across sectors on the rate of profit, with capital coefficients remaining unchanged. But in the

[2] In terms of the alternative proof given in footnote 1, this follows directly from the fact that when ε =1, (1-k-km(t)) = 0, in which case, as discussed there, the rate of profit will remain unchanged even with unequal labour productivity growth rates across sectors.

case where capital coefficients change but labour productivity growth is equal across sectors, there is no such divergence of results arising from the value of ε. This is shown below:

Lemma 2: If labour productivity grows at the same rate across sectors, but the condition a=a', b=b' does not hold, then r cannot remain unchanged.

Proof: If labour productivity grows at the same rate across sectors then wl_i remains unchanged. Now, suppose a<a' while b=b'. From the second price equation (v), p remains unchanged if r is unchanged. But for the first commodity then $a'r + w'.l_1.p > 1$, which violates the price equation (iv). Therefore r cannot remain unchanged.

Likewise if a=a' while b<b', then r can remain unchanged only if p remains unchanged, but if r is unchanged, then p which equals $b'.r/(1-w'l_2)$ must be higher than before. Hence r cannot remain unchanged. And if a<a', then for r to remain unchanged p must fall which is impossible as long as b≤b'. It follows that r can not remain unchanged.

Q.E.D.

This lack of divergence arising from the value of ε disappears once more the moment we allow for the possibility of unequal labour productivity growth across sectors. Let us start with the case $\varepsilon = 1$.

Lemma 3: If either or both a and b increase because of technological progress, then the rate of profit cannot remain unchanged, no matter what the pattern of labour productivity increase across sectors, if real wages rise in tandem with the weighted average labour productivity (as in (T)) and if savings equal profits.

Proof: Adding the price formation equations (iv) and (v), we get

$$r(a+b) + p.\sum w.l_i = 1 + p$$

$$\text{or,} \qquad r(a+b) + p(\sum w.l_i - 1) = 1. \qquad \text{(vii)}$$

Since savings and profits are equal, the second term on the LHS is zero, in which case if either or both a and b rise because of technological progress, i.e. (a+b) increases, then r can not remain unchanged; it must fall.

Q.E.D.

For the case where $\varepsilon < 1$, however, we can only have the following Lemma.

Lemma 3': If either or both a and b increase because of technological progress, then the rate of profit cannot remain unchanged, no matter what the pattern of labour productivity increase across sectors, provided it rises no slower in sector 2 than in sector 1, if real wages rise in tandem with the weighted average labour productivity (as in (T)) and if savings are less than profits.

Proof: Labour productivity rising no slower in sector 2 than in sector 1, implies that $w'l'_2 \leq wl_2$, which means that $wl_1 \geq w'.l_1'$. The case where the equality sign holds has already been discussed in Lemma 2. We are concerned here therefore with the strict inequality case.

For r to remain unchanged when either a or b or both rise, it follows from (iv) that p must fall (since wl_1 rises). But, an unchanged rate of profit with a lower p must mean a larger relative size of the consumption goods sector, i.e. a change in the sectoral capital stock ratios in favour of this sector, which necessarily means a lower growth rate. Since when $\varepsilon<1$, a fixed ratio of profits is saved (say s_p), $g = r.s_p$, a fall in g must therefore entail a fall in r, i.e. r cannot remain unchanged.

Q.E.D.

In view of the above we can advance the following two theorems for our two cases:

Theorem 2: If technological progress entails that $a \leq a'$, $b \leq b'$, and $l_i \leq l'_i$ with at least one of the l_is declining, and l_2 declining no slower than l_1, and if savings< profits, and wages rise with productivity as in (T), then the following constitute necessary and sufficient conditions for the rate of profit to remain unchanged by technological progress: (1) a' = a, and b' = b, and (2) the growth of labour productivity is same in both sectors.

Proof: Sufficiency has already been shown in Theorem 1.
Necessity can be shown as follows. Cases of non-fulfilment of conditions (1) and (2) can fall into three groups: (i) where a=a', b=b' but growth of labour productivity is unequal across sectors; (ii) where growth of labour producity is equal across sectors but either a'>a or b'>b or both; and (iii) where labour productivity growth is unequal across sectors and either a'>a or b'>b or both. Lemmas 1, 2 and 3 establish respectively that r cannot remain unchanged in cases (i), (ii) and (iii). It follows that for r to remain unchanged both (1) and (2) must hold.

Q.E.D.

Theorem 2': If technological progress entails that $a \leq a'$, $b \leq b'$, and $l_i \leq l'_i$ with at least one of the l_is declining, and if savings= profits, and wages rise with productivity as in (T), then the following constitute necessary and sufficient conditions for the rate of profit to remain unchanged by technological progress: a' = a, and b' = b.

Proof: When savings equal profits, $\varepsilon=1$ and (vii) simply becomes $r(a+b) = 1$. Both necessity and sufficiency follow from this.

Q.E.D.

From Theorems 2 and 2', it is easy to move ahead and establish the following theorem.

Theorem 3: In the universes described by Theorems 2 and 2', if the respective necessary and sufficient conditions do not hold, then the rate of profit falls as a consequence of technological progress.

Proof: In the case where $\varepsilon = 1$, this is obvious. In the case of $\varepsilon<1$, i.e. the domain of Theorem 2, it can be established as follows. Suppose condition (1) holds but not (2). Then $w.l_1 > w'.l_1'$, and from (iv) that a rise in r is possible only if p falls. Lemma 3 however shows that a fall in p must be associated with a fall in r. Hence r cannot rise if (1) holds but not (2).

Likewise if (2) holds but not (1), then r can rise only with a fall in p, which from Lemma 3 must be associated with a fall in r. Hence r cannot rise if (2) holds but not (1).

Finally if neither (1) nor (2) holds, i.e. technological progress causes labour productivity growth to be unequal across sectors, with l_2 falling no slower than l_1, *and* also raises some capital input(s), then again from Lemma 3 r cannot rise.

It follows that r cannot rise when (1) and (2) do not both hold. Since it cannot remain unchanged according to Theorem 2, it follows that r must fall.

Q.E.D.

4. Comments and conclusions

What is striking about the above results is that the effect of technological progress on the rate of profit appears to depend, apart from the usual factors, *also upon savings behavior out of profits.*

The intuitive reason for this is the following. When a part of profits is consumed, then a change in the price of the consumer good must be associated with a change in the size of the consumption sector, i.e. in the deployment of capital stock between the two sectors, *even if the rise in real wages in tandem with productivity leaves workers' demand for the consumption good unchanged in the wake of technological progress in the first instance.* And a change in the deployment of capital stock must affect the growth rate, and hence by inference the rate of profit. This is quite different from what one may, if one is incautious, expect on the basis of the Sraffa system (since, with wages being paid at the end of the period, the consumption good here may appear to be a "non-basic" in Sraffa's sense, with the size of its output incapable of affecting the rate of profit).

What is also striking is that where savings equal profits, then *no matter what the pattern of labour productivity growth across sectors*, the rate of profit must fall if either or both of the capital coefficients rise. Where savings are less than profits, this may not happen if labour productivity rises faster in the capital goods sector; but if the opposite is the case, i.e. labour productivity rises faster in the consumption goods sector, then the rate of profit must fall in this case too, no matter whether capital coefficients rise or not and in what pattern across sectors.

For the rate of profit to remain unchanged, when savings differ from profits, and when labour productivity grows no slower in the consumption sector, a set of strict conditions has to be fulfilled. If this restriction, of labour productivity growing no slower in the consumption goods sector, is not there, then these strict conditions, though sufficient, are not necessary: even in their absence the rate of profit may remain unchanged, but only through fortuitous coincidences.

Going back to the case where savings equal profits, i.e. where the classical savings assumption holds, the fact that technological progress being more rapid in the capital goods sector still does not prevent a fall in the rate of profit if the capital coefficient increases *in any sector*, has extremely far-reaching significance. While Marx's view that technological progress consists in the substitution of dead for living labour may be generally accepted, the argument against his prognostication of a falling tendency of the rate of profit has been that the organic composition in *value terms* may not increase, since faster labour productivity growth in the capital goods sector will cheapen capital goods. What the above argument shows is that this is not correct. *Even if labour productivity grows faster in capital goods production, a rise in capital coefficient in any sector will still lower the rate of profit*[3].

It may of course be argued that our results are affected by the assumption that wages are fixed in consumer goods and rise with the weighted average labour productivity. We do not say anything about

[3] The only general proposition one can make about the *pattern of labour productivity growth across sectors* affecting the direction of movement of the rate of profit is for the case where savings are less than profits, and that too for an exceptional situation. And that statement is the following. *If technological progress leaves the capital coefficients unchanged, i.e. $a = a'$ and $b = b'$, and if savings are less than profits, then the rate of profit will rise when labour productivity rises faster in the capital goods sector, and fall when it rises faster in the consumption goods sector.* This can be proven on the basis of Lemma 3 (though I shall not do so here), but it holds only in the special case where capital coefficients remain unchanged. When they do change, no general conclusion can be unambiguously affirmed.

the share of wages in value added remaining constant between the pre-and post-technological progress situations. But no behaviour can be adduced to sustain the latter postulate.

Let us imagine an economy that is experiencing steady state growth on the basis of given techniques. Now suppose there is one-shot technological progress which changes labour and capital coefficients in both sectors. On the basis of wage behaviour as postulated in the above model, which has a certain real basis, if the rate of profit falls, then the economy gets displaced from its equilibrium path; how it moves in its disequilibrium state, whether it ever regains equilibrium are matters on which we can say nothing. Hence to argue that the workers would bargain to obtain a given share of the value added at the new equilibrium prices, whose establishment itself is dubious, makes little sense. If equilibrium cannot be sustained on the basis of the postulated wage behaviour, then it simply cannot be sustained.